

Formulas for free: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

1. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = 2x^2 - 5x + 1$.
[You use should the rules we learned to double check your answer.]

Remember to find $f(\text{something})$ we replace every occurrence of x with our something. We have

$$f\left(\boxed{}\right) = 2\left(\boxed{}\right)^2 - 5\left(\boxed{}\right) + 1$$

So $f(x+h) = 2(x+h)^2 - 5(x+h) + 1$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 5(x+h) + 1) - (2x^2 - 5x + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 1 - 2x^2 + 5x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4x + 2h - 5)}{h} \\ &= \lim_{h \rightarrow 0} 4x + 2h - 5 \\ &= 4x + 0 - 5 \end{aligned}$$

So $f'(x) = 4x - 5$ (which is exactly what our “rules” tell us.)

2. Given y , compute its derivative y' . Don't worry about simplifying your answers.

(a) $y = x^3e^x + 2x^5 + 1234$

$$y' = (3x^2e^x + x^3e^x) + 10x^4 + 0$$

(b) $y = \frac{\ln(x)}{3x-6}$

$$y' = \frac{(1/x)(3x-6) - \ln(x)(3)}{(3x-6)^2}$$

(c) $y = \sqrt{x^2+1}$

Rewrite y a little as $y = (x^2+1)^{1/2}$. Then use the chain rule with $y = u^{1/2}$ and $u = x^2+1$. (We are using a special case of the chain rule called the “generalized power rule”.)

$$y' = \frac{1}{2} (x^2+1)^{-1/2} (2x+0)$$