1. $\displaystyle\int 3x^2 - \frac{1}{x} + e^x - 7\,dx = x^3 - \ln|x| + e^x - 7x + C$

2. $\displaystyle\int \frac{\sqrt{x}}{x^3} + \frac{6+2x}{x^2} - 50x^{99}\,dx = \int x^{-3+1/2} + \frac{6}{x^2} + \frac{2x}{x^2} - 50x^{99}\,dx$
$\displaystyle = \int x^{-5/2} + 6x^{-2} + \frac{2}{x} - 50x^{99}\,dx = \frac{x^{-3/2}}{-3/2} + 6\frac{x^{-1}}{-1} + 2\ln|x| - 50\frac{x^{100}}{100} + C$
$\displaystyle = -\frac{2}{3x^{3/2}} - \frac{6}{x} + 2\ln|x| - \frac{1}{2}x^{100} + C$

3. $\displaystyle\int dx = x + C$

4. $\displaystyle\int \frac{e^{5x}}{e^{2x}} - 8x^{7/3}\,dx = \int e^{5x-2x} - 8x^{7/3}\,dx = \int e^{3x} - 8x^{7/3}\,dx$
$\displaystyle = \frac{1}{3}e^{3x} - 8\frac{x^{10/3}}{10/3} + C = \frac{e^{3x}}{3} - \frac{24}{10}x^{10/3} + C$

5. $\displaystyle\int (x^2-5x+6)^9(2x-5)\,dx$ Substitute $u = x^2 - 5x + 6$ so that $du = (2x-5)\,dx$. So our integral becomes $\displaystyle\int u^9\,du = \frac{u^{10}}{10} + C = \frac{(x^2-5x+6)^{10}}{10}$

6. $\displaystyle\int e^{6x-2}\,dx$ Substitute $u = 6x-2$ so that $du = 6\,dx$ thus $(1/6)\,du = dx$. So our integral becomes $\displaystyle\int e^u \frac{1}{6}\,du = \frac{1}{6}e^u + C = \frac{e^{6x-2}}{6} + C$

7. $\displaystyle\int \frac{x+3}{x^2+6x-7}\,dx$ Substitute $u = x^2+6x-7$ so that $du = (2x+6)\,dx = 2(x+3)\,dx$ thus $(1/2)\,du = (x+3)dx$. So our integral becomes $\displaystyle\int \frac{(1/2)\,du}{u} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$
$\displaystyle = \frac{1}{2}\ln|x^2+6x-7| + C$

8. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx = \int e^{x^{-1}}x^{-2}\,dx$ Substitute $u = x^{-1}$ so that $du = -x^{-2}\,dx$ thus $-du = x^{-2}\,dx$. So our integral becomes $\displaystyle\int e^u(-du) = -e^u + C = -e^{1/x} + C$

9. $\int \frac{\ln(x)}{x}\,dx = \int \ln(x)\frac{1}{x}\,dx$ Substitute $u = \ln|x|$ so that $du = (1/x)\,dx$. So our integral becomes $\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln|x|)^2 + C$

10. $\int e^{e^x} e^x\,dx$ Substitute $u = e^x$ so that $du = e^x\,dx$. So our integral becomes $\int e^u\,du = e^u + C = e^{e^x} + C$

Suppose a company's marginal profit is approximated by $MP(q) = 30q - 5$ and we know that they have a break even point when $q = 10$. Approximate their profit when $q = 50$.

Roughly – marginal profit measures the change in profit as does the derivative. So let's assume $P'(q) = MP(q) = 30q{+}5$. Then $P(q) = \int 30q + 5\,dq = 30\frac{q^2}{2} + 5q + C = 15q^2 + 5q + C$ We know that $P(10) = 0$ (since $q = 10$ is a break even point). So $0 = P(10) = 15(10^2) + 5(10){+}C$ which means $0 = 1550{+}C$ so $C = -1550$. Therefore, the company's profit function is (approximately) $P(q) = 15q^2{+}5q{-}1550$. Finally, $P(50) = 15(50^2){+}5(50){-}1550 = 36200$.

Answer: Their profit is approximately \$36,200 when $q = 50$.

Find the antiderivative $f(x)$ of $g(x) = \dfrac{500}{\sqrt{2x+1}} + 6x^2 + 3$ such that $f(4) = 10$.

$f(x) = \int g(x)\,dx = \int 500(2x+1)^{-1/2} + 6x^2 + 3\,dx = 500(2x+1)^{1/2} + 2x^3 + 3x + C$ (For the term "$500(2x+1)^{-1/2}$" use the substitution $u = 2x + 1$). We also know that $f(4) = 10$ so $10 = f(4) = 500(2(4)+1)^{1/2} + 2(4^3) + 3(4) + C$ $10 = 500\sqrt{9} + 128 + 12 + C$ so $10 = 1500 + 140 + C$ so $C = -1630$.

Answer: $f(x) = 500\sqrt{2x+1} + 2x^3 + 3x - 1630$