Math 1030 **Workshop #3** **June 7^{th}, 2010**

Name: ____________________________ **Due: Thursday, June 11^{th}**

1. Quentin owns a rental car store in a Tinytown, NC. Instead of running a franchise for a national chain, Quentin just acts as a middleman. Since Tinytown is so small and remote, it can be prohibitively expensive for Quentin to aquire a large number of rental cars. Here is some supply and demand data Quentin has collected:

Number of Cars Rented	1	10	50	100	150
Rental Price	\$500	\$225	\$125	\$75	\$30
Quentin's Cost Per Car	\$50	\$15	\$40	\$120	\$300

So, for example, when Quentin charged \$225 to rent a car for the day, he had 10 customers rent from him and was able to obtain (from various suppliers) 10 rental cars at a price of \$15 per car per day.

(a) Use the table of data to find supply and demand price functions (use rental prices for demand and Quentin's cost for supply). Use a logarithmic model for the demand function and a quadratic for the supply function.

Demand function: $p_d =$ ____________________________

Supply function: $p_s =$ ____________________________

The market equilibrium is $(q_E, p_E) = \Big($ __________ , __________ $\Big)$.

If Quentin charges \$100 to rent a car for the day, about how many customers should he expect? ______________ customers.

(b) Use your model for the demand function to find a revenue function. Use your supply function to model the variable cost per car and the fact that Quentin has fixed costs of \$500 per day to find Quentin's cost function. Finally, use your revenue and cost functions to find profit, marginal revenue, marginal cost, and marginal profit functions.

Quentin has _____ break even points.

These occur when Quentin rents ____________________________ cars.

Quentin's 50^{th} rental car costs him \$______________ per day.

Quentin maximizes his profit when he rents to __________ customers.

The corresponding optimal rental price is \$______________ per car per day.

Quentin's maximum possible profit is \$______________ per day.

Notice that $\text{MC}(q) \neq 0$ for any q. Still the cost function **does** have a minimum.

How many rentals minimizes Quentin's costs? __________ cars.

2. Use Excel to compute the following limits. If the limit does not exist write "DNE".

   (a) $\lim_{x \to 2} \dfrac{x^3 + x^2 - 5x - 2}{x^2 - 4} =$ ____________________

   (b) $\lim_{x \to 0} \text{atan}\left(\dfrac{1}{x}\right) =$ ____________________

   Note: "atan" is the inverse tangent function. In Excel, "=ATAN(A1)" would compute the inverse tangent of the value in cell A1.

3. Recall that the **derivative** of $f(x)$ is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   where $\dfrac{f(x+h) - f(x)}{h}$ is called the **difference quotient** of $f(x)$.

   Let $f(x) = e^{-x^2}$. Compute the difference quotient of $f(x)$ when $x = 0.5$ and $h = 2$.

   $\dfrac{f(0.5+2) - f(0.5)}{2} =$ ____________________

   Now use Excel to compute the limit as $h \to 0$. This shows that $f'(0.5) =$ ________________.

   Finally, redo these calculations when $x = -1$.

   $\dfrac{f(-1+2) - f(-1)}{2} =$ ____________________

   $f'(-1) =$ ____________________