

1. Compute the derivative of  $f(x) = x^2 + 2x + 3$  using the limit definition of the derivative:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{((x+h)^2 + 2(x+h) + 3) - (x^2 + 2x + 3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 3 - x^2 - 2x - 3}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h + 2)}{h} = \lim_{h \rightarrow 0} 2x + h + 2 = 2x + 2 \end{aligned}$$

2. Find the derivative of  $y = x^2 \ln(x) + (1 + e^x)^{100}$

Use the product rule for the first term and the generalized power rule (or chain rule) for the second term.

$$y' = 2x \ln(x) + x^2 \frac{1}{x} + 100(1 + e^x)^{99} e^x = 2x \ln(x) + x + 100(1 + e^x)^{99} e^x$$


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1. Compute the derivative of  $f(x) = x^2 + 4x - 1$  using the limit definition of the derivative:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{((x+h)^2 + 4(x+h) - 1) - (x^2 + 4x - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 1 - x^2 - 4x + 1}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 4h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h + 4)}{h} = \lim_{h \rightarrow 0} 2x + h + 4 = 2x + 4 \end{aligned}$$

2. Find the derivative of  $y = e^x \ln(x) + \sqrt{3x+1}$  ( $= e^x \ln(x) + (3x+1)^{1/2}$ )

Use the product rule for the first term and the generalized power rule (or chain rule) for the second term.

$$y' = e^x \ln(x) + e^x \frac{1}{x} + \frac{1}{2}(3x+1)^{-1/2}(3)$$