Math 1030 Workshop #4 (June 10, 2009) **Name:** <u>**ANSWER KEY**</u>

DUE: Tuesday, June 15^{th} Please turn in a paper copy and **SHOW YOUR WORK!**

1. Use the limit definition of the derivative to find $f'(x)$ if...
   [You use should the rules we learned to double check your answer.]

   (a) $f(x) = 3x^2 - x + 8$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(3(x+h)^2-(x+h)+8)-(3x^2-x+8)}{h} \\ &= \lim_{h\to 0} \frac{3x^2+6xh+h^2-x-h+8-3x^2+x-8}{h} = \lim_{h\to 0} \frac{6xh+h^2-h}{h} \\ &= \lim_{h\to 0} \frac{h(6x+h-1}{h} = \lim_{h\to 0} 6x+h-1 = 6x-1 \end{aligned}$$

   (b) $f(x) = \dfrac{1}{x^3}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{x^3}{(x+h)^3x^3}-\frac{(x+h)^3}{x^3(x+h)^3}}{h} = \lim_{h\to 0} \frac{\frac{x^3-(x+h)^3}{x^3(x+h)^3}}{\frac{h}{1}} \\ &= \lim_{h\to 0} \frac{x^3-(x^3+3x^2h+3xh^2+h^3)}{x^3(x+h)^3h} = \lim_{h\to 0} \frac{-3x^2h-3xh^2-h^3)}{x^3(x+h)^3h} \\ &= \lim_{h\to 0} \frac{h(-3x^2-3xh-h^2)}{x^3(x+h)^3h} = \lim_{h\to 0} \frac{-3x^2-3xh-h^2}{x^3(x+h)^3} \\ &= \frac{-3x^2-0-0}{x^3(x+0)^3} = \frac{-3x^2}{x^6} = \frac{-3}{x^4} \end{aligned}$$

2. Find the equation of the line tangent to the graph of $y = f(x)$ at $x = x_0$ if...

   (a) $f(x) = 3x^2 - x + 8$ and $x_0 = 1$

   We need a point and a slope to find the equation of the tangent line. If $x = 1$, then $y = 3(1^2) - 1 + 8 = 10$. So our line passes through the point $(1, 10)$.
   The slope of the tangent is given by the derivative. $f'(x) = 6x - 1$, so the slope is $m = f'(1) = 6(1) - 1 = 5$.
   Using point-slope we find that $y - 10 = 5(x - 1)$ and so...

   Answer: $y = 5x + 5$

   (b) $f(x) = \dfrac{1}{x^3}$ and $x_0 = -1$

   We need a point and a slope to find the equation of the tangent line. If $x = -1$, then $y = 1/(-1)^3 = -1$. So our line passes through the point $(-1, -1)$.
   The slope of the tangent is given by the derivative. $f'(x) = -3/x^4$, so the slope is $m = f'(-1) = -3/(-1)^4 = -3$.
   Using point-slope we find that $y - (-1) = -3(x - (-1))$ and so...

   Answer: $y = -3x - 4$

3. Compute the derivative of each of the following functions. Please simplify your answers.

(a) $y = 10\ln(x) - 6e^x + \sqrt[3]{x} + \frac{1}{x} + 9x - 42$

We should simplify first, then this one's easy.
$y = 10\ln(x) - 6e^x + x^{1/3} + x^{-1} + 9x - 42$

Answer: $y' = \dfrac{10}{x} - 6e^x + \dfrac{1}{3}x^{-2/3} + 9$

(b) $y = \ln(2x+1)e^x$

We need to use the product rule with parts: $\ln(2x+1)$ and e^x. In addition, to take the derivative of the first part, we'll need to use the chain rule: $y = \ln(u)$ and $u = 2x+1$ so $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \dfrac{1}{u}(2) = \dfrac{2}{2x+1}$

Answer: $\dfrac{2}{2x+1}e^x + \ln(2x+1)e^x$

(c) $y = \dfrac{x^2e^x + 2}{x^2+x+1}$

This one requires the quotient rule. Also, we need the product rule to find the derivative of the top.

Answer: $y' = \dfrac{(2xe^x + x^2e^x)(x^2+x+1) - (x^2e^x+2)(2x+1)}{(x^2+x+1)^2} = \dfrac{(x^4+x^3+2x^2+2x)e^x - 4x - 2}{(x^2+x+1)^2}$

(d) $y = (xe^x+1)^{25}$

We have xe^x+1 sitting "inside" of the power function u^{25}. So we'll use the generalized power rule (a special case of the chain rule). To find the derivative of xe^x+1 we'll need to use the product rule.

Answer: $y' = 25(xe^x+1)^{25}(e^x + xe^x)$

(e) $y = \ln\left(\dfrac{(5x+1)e^{10x}}{x^3}\right)$

This function is best handled by first using laws of logs to split it apart. We get $y = \ln((5x+1)e^{10x}) - \ln(x^3) = \ln(5x+1) + \ln(e^{10x}) - 3\ln(x) = \ln(5x+1) + 10x - 3\ln(x)$ Notice that $\ln(5x+1)$ can't be split up anymore (because its arguments are added – not multiplied, divided, or exponentiated). To handle $\ln(5x+1)$ we'll need to use the chain rule.

Answer: $y' = \dfrac{5}{5x+1} + 10 + \dfrac{3}{x}$