Math 1030 Workshop #5 (June 15, 2009) **Name: <u>ANSWER KEY</u>**

DUE: Friday, June 18$^{\text{th}}$ Please turn in a paper copy and **SHOW YOUR WORK!**

1. Consider the function $P(q) = \begin{cases} -2q & q \leq 0 \\ (q-0.25)^2 e^{-0.36(q-0.25)^2} & 0 < q < 2 \\ 2q^3 - 10.6q^2 + 17.6q - 8.8 & q \geq 2 \end{cases}$

   Be careful! Wolfram Alpha has a hard time interpreting commands applied to this function. You may want to deal with the function one piece at a time.

   (a) Find all of the critical points of $P(q)$. $q = 0, 0.25, 1.917, 2, 2.2$

   Correctly typed in we should have:

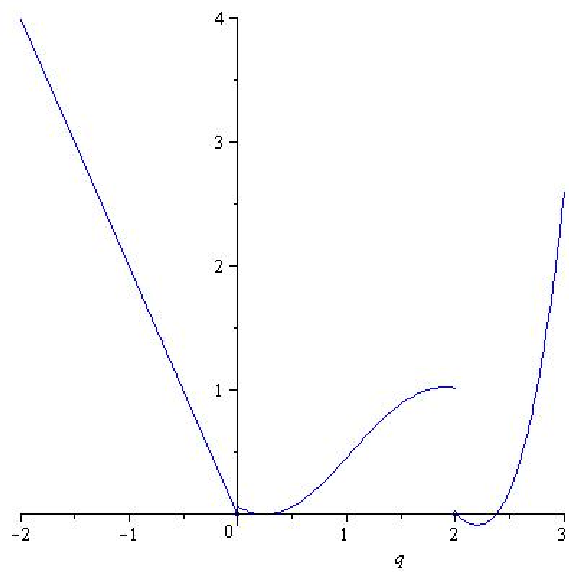
   "`Piecewise[{ {-2q,q<=0}, {(q-0.25)^2 e^(-0.36(q-0.25)^2),0<q<2}, {2q^3-10.6q^2+17.6q-8.8,q>=2} }]`"

   But Alpha chokes on this input. So we must handle this function *piecewise* — one piece at a time.

   First, the derivative of $-2q$ is -2. Since $-2 \neq 0$, we have no critical points for $q < 0$. Now $q = 0$ is a critical point because we have a discontinuity at $q = 0$ and thus the derivative does not exist there.

   Moving on to the second piece, we must take the derivative and find everywhere it is either undefined or equal to 0. In Alpha we type: "`derivative of (q-0.25)^2 e^(-0.36(q-0.25)^2)`" This derivative exists everywhere (we don't have any division by zero, logarithm, or square root problems) and Alpha finds 3 roots: $q = -1.417$, $q = 0.25$, and $q = 1.917$. Remember that this formula is only good when $0 < q < 2$, so the first root is "out of bounds". The other 2 roots are ok. So we have $P'(q) = 0$ when $q = 0.25$ and $q = 1.917$ (2 more critical points). Next, consider $q = 2$. We have yet another discontinuity so $q = 2$ is critical as well.

   Moving on the the third and final piece, we must take its derivative and repeat the process. In Alpha we type: "`derivative of 2q^3-10.6q^2+17.6q-8.8`". Yet again this derivative exists everywhere. This time we have 2 roots: $q = 1.333$ and $q = 2.2$. The third formula is only valid for $q \geq 2$, so $q = 1.333$ is "out of bounds". Thus we get one last critical point at $q = 2.2$.



A plot $P(q)$ on the interval $[-3, 2]$.

This plot was generated in Maple using the following commands:

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[> P := q -> piecewise(q<=0,-2*q,0<q and q<2,(q-0.25)^2*exp(-0.36*(q-0.25)^2),
              q>=2,2*q^3-10.6*q^2+17.6*q-8.8);
[> plot(P(q),q=-2..3,discont=true,color=blue);
```

(b) Restricting to the interval $[-2, 3]$ find the minimum and maximum values and their locations.

We have already found the critical points. To determine mins and maxs, we must examine what happens to the function at these points as well as the endpoints. So we need to plug $-2, 0, 0.25, 1.917, 2, 2.2, 3$ into our function. *Note:* Technically we need to check left and right hand limits as well as function values at discontinuous points, but we gloss over this technicality since it doesn't effect our answer.

First, plug in -2 and 0 into $-2q$ (since this is the formula we use for values of q less than or equal to 0). $-2(-2) = 4$ and $-2(0) = 0$.

Next, we use the second formula to evaluate points between 0 and 2. So in Alpha we type: "`(q-0.25)^2 e^(-0.36(q-0.25)^2) where q=0.25`" etc. To save time we can have Alpha plug in several values at once: "`(q-0.25)^2 e^(-0.36(q-0.25)^2) where q={0.25,1.917}`" We get $P(0.25) = 0$ and $P(1.917) = 1.022$.

Finally, we need to plug in all of the point where $q \geq 2$ into the third formula. In Alpha we type: "`2q^3-10.6q^2+17.6q-8.8 where q={2,2.2,3}`" We get $P(2) = 3.55271 \times 10^-15$, $P(2.2) = -0.088$, and $P(3) = 2.6$. Notice that $P(2) \approx 0$.

Summing all of this up, we have found:

$q =$	-2	0	0.25	1.917	2	2.2	3
$P(q) =$	4	0	0	1.022	0	-0.088	2.6

The maximum value of $P(q)$ is 4. This occurs when $q = -2$.

The minimum value of $P(q)$ is -0.088. This occurs when $q = 2.2$.

2. Wendy's small business uses a moderately expensive small copier. It costs \$1,500 to purchase a new copier and she has collected the following repair cost data: The first year's repair costs were \$75 and the second year's repairs cost \$325. Model the average annual cost of the copier using a function of the form: $A(t) = \frac{C}{t} + Rt^r$ where C is the cost of purchasing a copier and Rt^r models the repair costs.

Use the facts $Rt^r = 75$ when $t = 1$ and $Rt^r = 200 \left(= \frac{75 + 325}{2}\right)$ when $t = 2$ to find R and r.

To find $A(t)$ we need to determine C, R, and r. We are told that $C = 1500$. To find R and r, we'll need to use our "facts". First, $Rt^r = 75$ when $t = 1$. This says that $R(1^r) = 75$. But $1^r = 1$ (for any r) so $R = 75$. The second fact says that $Rt^r = 200$ when $t = 2$. Plugging in what we know, we have $75(2^r) = 200$. It is not too hard to solve this equation by hand (use logarithms). However, it's even easier to make Alpha do it. Type: "`75(2^r)=200`" into Alpha and find that $r = \frac{\ln(8/3)}{\ln(2)} \approx 1.415$.

$$A(t) = \frac{1500}{t} + 75t^{1.415}$$

Next, we need to minimize $A(t)$. If we type "`plot 1500/t + 75t^1.415 on [0,5]`" in Alpha we get a nice plot which shows us that the minimum is located somewhere near $t = 3$. To find the exact location of the minimum we need to find the critical points of $A(t)$. In Alpha type: "`derivative 1500/t + 75t^1.415`". The derivative is undefined at $t = 0$ (but this obviously isn't a minimum) and has a root when $t = 2.99436$ (right where we expected the minimum to be). We need to plug this into $A(t)$ to find the minimum cost. Type: "`1500/t + 75t^1.415 at t=2.99436`" and get \$854.97. Finally, we need to convert $t = 2.99436$ into years and months. We have 2 years plus 0.99436 years. Type "`0.99436 years`" in Alpha and Alpha will tell you that this is 11.9323 months or about 12 months. So our costs are minimized if we buy a new copier every 3 years.

Wendy should replace the copier every 3 years and 0 months.

If she does this, her average annual cost will be \$854.97.

3. Frank has a neighborhood grocery store. One of his most popular items is 25 lbs. bags of premium rice. In fact, he sells 1000 bags each year. Each bag costs Frank \$20. He pays \$150 every time he gets the rice delivered and his inventory costs are \$1.25 per bag per year (base inventory on average inventory with the standard assumptions).

   (a) $C(x) = 1000(20) + 1.25\left(\frac{x}{2}\right) + 150\left(\frac{1000}{x}\right)$

   First, let's type "`plot 1000(20)+1.25(x/2)+150(1000/x) on [10,1000]`" in Alpha. I chose the domain "$[10, 1000]$" to cut-off the spike in cost that comes from ordering too few bags at a time. This plot shows that the minimum cost occurs somewhere around 500 bags. To find the exact answer, we need to find the critical points of $C(x)$. In Alpha type "`derivative 1000(20)+1.25(x/2)+150(1000/x)`". The derivative is undefined at $x = 0$ (which is obviously not the minimum) and the derivative is zero (has roots) when $x = \pm 489.898$. Throwing out the negative answer, we find that our minimum occurs at 489.898 (500 wasn't a bad guess). We need to plug this into $C(x)$ to find the corresponding minimal cost. Type: "`1000(20)+1.25(x/2)+150(1000/x) at x=489.898`" and get \$20612.40.

   Frank's **ideal** EOQ is 489.898. His **ideal** minimum annual cost is \$20,612.40.

   (b) Suppose that Frank gets a discount if he places an order of 400 or more bags of rice. For orders of 400 or more, he gets the bags for \$17.50. However, his shipping costs increase to \$200 for a large shipment and he found that his inventory cost rise to \$2.50 a bag.

   $$C(x) = \begin{cases} 1000(20) + 1.25\left(\frac{x}{2}\right) + 150\left(\frac{1000}{x}\right) & x < 400 \\ 1000(17.50) + 2.50\left(\frac{x}{2}\right) + 200\left(\frac{1000}{x}\right) & x \geq 400 \end{cases}$$

   Type: "`Piecewise[{ {1000(20)+1.25(x/2)+150(1000/x),x<400}, {1000(17.50)+2.50(x/2)+200(1000/x),x>=400} }]`" into Alpha and Alpha will spit out a plot which clearly shows that the minimum occurs when $x \geq 400$. Let's just focus on that piece of the function. Type "`plot 1000(17.50)+2.50(x/2)+200(1000/x) on [400,1000]`". The minimum seems to occur right at $x = 400$. Let's make sure by looking for critical points of $1000(17.50) + 2.50(x/2) + 200(1000/x)$. Type: "`derivative 1000(17.50)+2.50(x/2)+200(1000/x)`" into Alpha. We find that this function's derivative is undefined at $x = 0$ (which is "out of bounds") and is zero when $x = \pm 400$. Throwing out the negative answer, we get that 400 is critical. The minimum must occur at either an endpoint or a critical point. 400 is both and obviously the location of the minimum. Finally, we need to plug this into our function to find the minimal cost. Remember to use the second formula: "`1000(17.50)+2.50(x/2)+200(1000/x) at x=400`" and get \$18500.

   In this case, Frank's **ideal** EOQ is 400. His **ideal** minimum annual cost is \$18,500.

   (c) Are the solutions from parts (a) or (b) **practical solutions**? Explain.

   No. Neither solution is practical. First, we obviously can't order 489.898 bags of rice. If we round to 490 bags, then we'll need to make 1000/490 orders per year – which just doesn't work. On the other hand, we can order 400 bags of rice, but if we did we would need to make $1000/400 = 2.5$ orders per year – again we have a problem, unless we allow our inventory to spill over from one year to the next.