A Revenue, Cost, & Profit Problem in Wolfram Alpha

Following the corresponding example in Excel, we will construct revenue, cost, and profit functions. Then we will answer a few questions including, "When is profit maximized?"

Each command is followed by a "✓". Click on the "✓" to go to Wolfram Alpha and execute the command.

Example: Suppose our demand and supply price functions are given by $p_d(x) = -0.0075x^2 + 400$ and $p_s(x) = 0.00025x^3 - 0.05x^2 + 0.25x + 315$. In addition, suppose that fixed costs are \$5,000 and we are to use the supply price function to determine our variable costs. We wish to find the revenue, cost, and profit functions. Then use these to do some computations.

First, let's do a few computations with the supply and demand price functions and find the market equilibrium. For example, if we wish to know what prices go with the quantity $x = 75$, we simply use the command:

`-0.0075x^2+400 and 0.00025x^3-0.05x^2+0.25x+315 at x = 75` ✓

This tells us that the demand price is $p_d(75) = \$357.81$ and the supply price is $p_s(75) = \$157.97$ when $x = 75$. Next, let's find out what quantity goes with a demand price of \$85.

`-0.0075x^2+400 = 85` ✓

Alpha says that $p_d(x) = \$85.00$ when $x = 204.939$ (or about 205 items). Before finding the market equilibrium, let's graph the supply and demand functions together:

`plot -0.0075x^2+400 and 0.00025x^3-0.05x^2+0.25x+315 where x is between 0 and 220` ✓

Now let set these functions equal to each other and find the market equilibrium.

`-0.0075x^2+400 = 0.00025x^3-0.05x^2+0.25x+315` ✓

`-0.0075x^2+400 at x=175.354` ✓

So Alpha solves the equation and finds the demand and supply are equal when $x = 175.354$. We then plug that into one of the price functions to find the corresponding equilibrium price. Thus we find that the market equilibrium is $(q_E, p_E) = (175.354, \$169.38)$. Alternatively we could have asked Alpha to find "intersection" of expression one and expression two and Alpha would find both coordinates at the same time.

Finally, let's turn to our revenue, cost and profit functions:

- $R(x) = p_d(x) \cdot x = (-0.0075x^2 + 400)x$
- $C(x) = (\text{Fixed Costs}) + (\text{Variable Costs})x = 5000 + (0.00025x^3 - 0.05x^2 + 0.25x + 315)x$
- $P(x) = R(x) - C(x) = (-0.0075x^2 + 400)x - (5000 + (0.00025x^3 - 0.05x^2 + 0.25x + 315)x)$

Let's compute the marginal profit at $x = 25$. To do this we need to compute $P(24)$ and $P(25)$. We can do this in Alpha as follows:

`(-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) where x=24` ✓

`(-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) where x=25` ✓

Alternatively, we can use $x = [24, 25]$ to tell Alpha to plug in a list of values.

```
(-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) where x=[24,25] ✓
```

Alpha, then outputs the required values and we subtract and find that $MP(25) = P(25) - P(24) = -\$2,464.84 - (-\$2,599.42) = \$134.58$.

Let's find our break even points. These occur when profit is \$0 (or we could set revenue and cost equal to each other). In Alpha, when we give it an expression, one of its automatic reactions is to set our expression equal to zero and solve. So we just need to enter our profit function and Alpha will do the rest:

```
(-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) ✓
```

Alpha, then tells us that the "roots" (i.e. zeros) of the expression are $x = 39.608$ and 171.756. These are our break even quantities. Let's finish this example by finding the maximum possible profit. We could do this by just telling Alpha to "maximize" as follows:

```
maximize (-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) ✓
```

This tells us that the maximum profit occurs when $x = 128.744$. Of course we should round to try and find a realistic answer:

```
(-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) at x=[128,129] ✓
```

The maximum possible profit is \$23,808.30 and this occurs when we sell 129 items. We could find the corresponding optimal item sale price by plugging $x = 129$ into $p_d(x)$. This gives the result $p_d(129) = \$275.19$.

One final note: Be careful about using "maximize" blindly. It doesn't always work. You should always double check that the answer you're getting is reasonable. If maximize doesn't do the trick, we could always do this part of the problem, step-by-step. First, we would need to find the derivative of the profit function:

```
derivative (-0.0075x^2+400)x - (5000+(0.00025x^3-0.05x^2+0.25x+315)x) ✓
```

Alpha automatically found a "real root" (i.e. where $P'(x) = 0$) located at $x = 128.744$. This is the only critical point for the profit function (it's derivative exists everywhere). So our maximum must be located here (as we already know).