Math 1030 Workshop #4 **Name:** Answer Key

DUE: Tuesday, July 26^{th} Please turn in a paper copy and **SHOW YOUR WORK!**

1. Use the limit definition of the derivative to find $f'(x)$ if...
[You use should the rules we learned to double check your answer.]

(a) $f(x) = x^3 - 4x + 1$ $\qquad f\left(\boxed{x+h}\right) = \left(\boxed{x+h}\right)^3 - 4\left(\boxed{x+h}\right) + 1$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^3 - 4(x+h)+1] - [x^3-4x+1]}{h} \\ &= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-4x-4h+1-x^3+4x-1}{h} \\ &= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-4h}{h} = \lim_{h\to 0} \frac{h(3x^2+3xh+h^2-4)}{h} \\ &= \lim_{h\to 0} 3x^2+3xh+h^2-4 = 3x^2+0+0-4 = \boxed{3x^2-4} \end{aligned}$$

(b) $f(x) = \dfrac{1}{2x-1}$ $\qquad f\left(\boxed{x+h}\right) = \dfrac{1}{2\left(\boxed{x+h}\right)-1}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)-1}\cdot\frac{2x-1}{2x-1} - \frac{1}{2x-1}\cdot\frac{2(x+h)-1}{2(x+h)-1}}{h} \\ &= \lim_{h\to 0} \frac{\frac{2x-1}{(2(x+h)-1)(2x-1)} - \frac{2(x+h)-1}{(2x-1)(2(x+h)-1)}}{h} = \lim_{h\to 0} \frac{\frac{2x-1-2(x+h)+1}{(2(x+h)-1)(2x-1)}}{(h/1)} \\ &= \lim_{h\to 0} \frac{2x-2x-2h}{(2(x+h)-1)(2x-1)h} = \lim_{h\to 0} \frac{-2h}{(2(x+h)-1)(2x-1)h} \\ &= \lim_{h\to 0} \frac{-2}{(2(x+h)-1)(2x-1)} = \frac{-2}{(2(x+0)-1)(2x-1)} = \boxed{\frac{-2}{(2x-1)^2}} \end{aligned}$$

We can double check our answer: $f(x) = 1/(2x-1) = (2x-1)^{-1}$ so $f'(x) = -1(2x-1)^{-2}(2)$ (using the chain rule or more specifically the generalized power rule). Thus $f'(x) = -2/(2x-1)^2$ (which matches our answer coming from the limit definition).

2. Find the equation of the line tangent to the graph of $y = f(x)$ at $x = x_0$ if...

(a) $f(x) = x^3 - 4x + 1$ and $x_0 = 1$

- We need a point: $x = 1$ so that $y = f(1) = 1^3 - 4(1) + 1 = -2$
- And a slope: $y' = 3x^2 - 4$ so that $y'\Big|_{x=1} = 3(1)^2 - 4 = -1$ (the derivative evaluated at $x = 1$ gives us the slope of the tangent at $x = 1$).
- We put this together using point-slope: $y - y_0 = m(x - x_0)$ so that $y - (-2) = -1(x - 1)$ which simplifies to $y + 2 = -x + 1$

Answer: $y = -x - 1$

(b) $f(x) = \dfrac{1}{2x-1}$ and $x_0 = 0$

- We need a point: $x = 0$ so that $y = f(0) = \dfrac{1}{2(0)-1} = -1$
- And a slope: $y' = \dfrac{-2}{(2x-1)^2}$ so that $y'\Big|_{x=0} = \dfrac{-2}{(2(0)-1)^2} = \dfrac{-2}{(-1)^2} = -2.$
- We put this together using point-slope and get: $y - (-1) = -2(x - 0)$ which simplifies to $y + 1 = -2x$

Answer: $y = -2x - 1$

3. Compute the derivative of each of the following functions. Please simplify your answers.

Note: **I've boxed in acceptibly simplified answers.**

(a) $y = \sqrt{x} + 7e^x - 21\ln(x) + \frac{1}{x^5} - 3x + 11$

$y = x^{1/2} + 7e^x - 21\ln(x) + x^{-5} - 3x + 11$

$$\boxed{y' = (1/2)x^{-1/2} + 7e^x - \frac{21}{x} + (-5)x^{-6} - 3 \quad \text{or} \quad y' = \frac{1}{2\sqrt{x}} + 7e^x - \frac{21}{x} - \frac{5}{x^6} - 3}$$

(b) $y = \ln(x)e^{3x+1}$ We will need to use the product rule (and then the chain rule on e^{3x+1}).

$$\boxed{y' = \frac{1}{x}e^{3x+1} + \ln(x)e^{3x+1}(3) \quad \text{or} \quad y' = \frac{e^{3x+1}}{x} + 3\ln(x)e^{3x+1}}$$

(c) $y = \dfrac{x^3 - x^2 + 4}{1 - xe^x}$ We need to use the quotient rule (also the product rule is needed to take the derivative of the denominator).

$$\begin{aligned} y' &= \frac{(3x^2 - 2x)(1 - xe^x) - (x^3 - x^2 + 4)((-1)e^x + (-x)e^x)}{(1 - xe^x)^2} \\ &= \frac{3x^2 - 2x - 3x^3e^x + 2x^2e^x + (-x^3 + x^2 - 4)(-e^x - xe^x)}{(1 - xe^x)^2} \\ &= \frac{3x^2 - 2x - 3x^3e^x + 2x^2e^x + x^3e^x - x^2e^x + 4e^x + x^4e^x - x^3e^x + 4xe^x}{(1 - xe^x)^2} \end{aligned}$$

$$\boxed{y' = \frac{3x^2 - 2x + 4e^x + 4xe^x + x^2e^x - 3x^3e^x + x^4e^x}{(1 - xe^x)^2}}$$

or

$$\boxed{y' = \frac{x(3x - 2) + (x^4 - 3x^3 + x^2 + 4x + 4)e^x}{(1 - xe^x)^2}}$$

(d) $y = (\ln(2x + 1) + 15)^{80}$ We will need the chain rule (specifically the generalized power rule) for the outer function $(\text{STUFF})^{80}$ and then we'll need to use the chain rule again on the function $\ln(\text{STUFF})$.

$$y' = (80)(\ln(2x + 1) + 15)^{79} \cdot \frac{1}{2x + 1}(2)$$

$$\boxed{y' = \frac{160 \cdot (\ln(2x + 1) + 15)^{79}}{2x + 1}}$$

(e) $y = \ln\left(\dfrac{e^{5x}\sqrt{x - 7}}{(x^2 + 5)^6}\right)$ We should use laws of logarithms to break the function apart. Then we will need to use the chain rule a few times.

$y = \ln(e^{5x}(x - 7)^{1/2}) - \ln((x^2 + 5)^6) = \ln(e^{5x}) + \ln((x - 7)^{1/2}) - 6\ln(x^2 + 5)$ and finally $y = 5x + (1/2)\ln(x - 7) - 6\ln(x^2 + 5)$. Now we can differentiate.

$$y' = 5 + \frac{1}{2} \cdot \frac{1}{x - 7}(1) - 6\frac{1}{x^2 + 5}(2x)$$

$$\boxed{y' = 5 + \frac{1}{2x - 14} - \frac{12x}{x^2 + 5}}$$