Name: ANSWER KEY **Math 1030 Quiz #4** (July 27th, 2011)

1. Bert's office uses a lot of staples. He can purchase a case of staples for \$25. It costs \$5 to place an order and Bert pays \$0.50 to inventory a case of staples (base inventory costs on average inventory making all of the standard assumptions). Finally, Bert's office needs 100 cases of staples each year. Let $C(x)$ be the annual cost function.

   Therefore, the annual cost function is... $$C(x) = 25(100) + 5\left(\frac{100}{x}\right) + 0.50\left(\frac{x}{2}\right)$$

   List **ALL** of the critical points of $C(x)$ including "irrelevant" critical points (points outside the domain of reasonable x values). Round each to 3 decimal places.

   In ALPHA type "derivative 25(100)+5(100/x)+0.50(x/2)" and Alpha will return $C'(x)$ and automatically find its roots. Also, notice that the derivative is undefined at $x = 0$, so this can be considered a critical point as well.

   Critical points: $x = -44.7214, \quad 0, \quad 44.7214$

   Of course of these 3 critical points only $x = 44.7214$ is within our domain of 1 to 100 cases ordered at a time. Checking these points (by plotting "on [1,100]" or plugging them in using "where x=[1,44.7214,100]") we see that $x = 44.7214$ is indeed the location of the minimum possible annual cost.

   Bert's **ideal** EOQ is $x = 44.7214$ and minimum annual cost is $C(x) = \$2,022.36$.

2. Let $f(x) = \begin{cases} -x^2 + x + 4 & x \leq 2 \\ x - 2 & x > 2 \end{cases}$ Sketch the graph of $y = f(x)$ where $-2 \leq x \leq 4$.

   In ALPHA we type: `plot piecewise[{{-x^2+x+4,x<=2},{x-2,x>2}}] on [-2,4]`

   $f(x)$ has $\boxed{2}$ critical points.
   [You can tell by just looking at the graph.]

   There is 1 critical point at the peak of the piece of the parabola and a second critical point coming from the discontinuity.

   Alternatively we could look for critical points by differentiating each piece of our function and finding roots of the derivatives. We would need to use the commands:

   `derivative -x^2+x+4` and `derivative x-2`

   The first piece of our function has a critical point at $x = 1/2$ (the peak) and the second piece of the function has no critical points. [Note: $x = 1/2 \leq 2$ so this point occurs within the domain of the first formula.]

   Therefore, we have 2 critical points: $x = 0.5$ and 2.