Math 1030 Workshop #6 **Name:** <u>ANSWER KEY</u>

DUE: Wednesday, August 3rd Please turn in a paper copy and **SHOW YOUR WORK!**

1. Our hardware store is selling TorqueMaster 3000 lawn mowers. We have collected the following sales data:

Mowers Sold	8	14	19	24
Price	\$225	\$175	\$145	\$110

(a) Compute elasticity (round to 3 decimals places):

Mowers Sold	8		14		19		24
Price	\$225		\$175		\$145		\$110
Elasticity		2.181818		1.616162		0.847176	××× ×××

Using a formula something like: "=-((C2-B2)*(B3+C3))/((C3-B3)*(B2+C2))" where quantities are in row 2 and prices are in row 3.

(b) Model this demand price (in Excel) and find the logrithmic trendline.

Of course, we graph the quantity/price data and add a logarithmic trendline.

$p_d(q) = -101.9\ln(q) + 440.01$

According to this model, we will sell <u>31</u> mowers if we set our price at \$90.

This answer was found by goal seeking the predicted price to be \$90 by changing the quantity. Alternatively, we could ask Alpha to solve "-101.9 ln(q) + 440.01 = 90"

When $p_d(q) = \$90$, our point elasticity is $\varepsilon =$ <u>0.883</u> (round to 3 decimal places).

`derivative -101.9ln(x)+440.01 where x=31.0264` ✓

which gives us -3.2843 so $\varepsilon = -(p/q)/(dp/dq) = -(90/31.0264)/(-3.2843) = 0.8832$
Alternatively, we could compute the general formula for point elasticity:

`derivative -101.9ln(x)+440.01` ✓ `-((-101.9ln(x)+440.01)/x)/(-101.9/x)` ✓

which gives us $\varepsilon = 0.00981354(-101.9\ln(x) + 440.01)$ and so, plugging in $q = 31.0.264$, we get

`0.00981354 (440.01-101.9 ln(x)) where x=31.0264` ✓

Again, $\varepsilon = 0.883$.

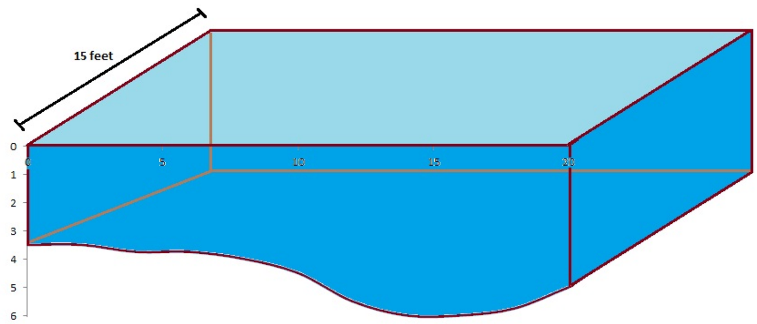
<u>Circle the correct answer:</u> We are currently charging \$90 for a lawn mower and want to **increase** revenue, we should [raise] / lower our price.

Since point elasticity (when $p = \$90$) is less than 1, we know that prices dominate and so revenue moves with price. Thus to increase revenue we should increase price.

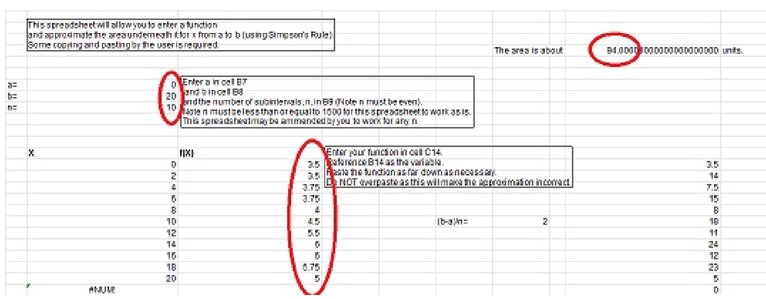
2. Local radio station WXYZ is getting ready for a contest where listeners will swim in a pool filled with Jello. The station manager has decided to host the contest in his own backyard and has measured the depth of his swimming pool at various points (measurements are in feet):

Distance from the end of the pool	0	2	4	6	8	10	12	14	16	18	20
Depth of the pool	3.5	3.5	3.75	3.75	4	4.5	5.5	6	6	5.75	5

Suppose that cross-sections of the manager's pool have constant depth and his pool is 15 feet wide. Help the manager determine the volume of Jello needed to fill the pool. Use the Simpson's rule Excel spreadsheet to determine the volume of the manager's pool (in cubic feet). Then convert your answer to gallons (use Alpha).



In the Simpson's rule Excel sheet, we enter $a = 0$, $b = 20$, $n = 10$ (we have split the interval $0 \le x \le 20$ into 10 pieces since we are counting by 2's). Then instead of typing in a formula, we simply manually punch in the data given to us.



The spreadsheet tells us that the area of the side of the pool is about 94 square feet. Thus the volume of the pool is approximately $94 \times 15 = 1410$ cubic feet. Typing "convert 1410 cubic feet to gallons" in Alpha gives us our answer of about 10,548 gallons.

The manager's pool will hold approximately ___1,410___ cubic feet of Jello.

This is approximately ___10,548___ gallons of Jello.

3. Flash-in-the-Pan Rock Inc. has released a new album by THE STENCH. They collected sales data and found that this album 3 months after its release it was selling at a rate of 10,000 albums a year. By the time the album has been out 6 months its sales rate went up to 20,000 albums a year. Assuming this data fits a curve of the form $S(t) = at^2e^{bt^2}$, use the sales data to solve for a and b.

We have been given the following information: $S(0.25) = 10000$ and $S(0.5) = 20000$ (after 3 and 6 months the rates of sales are 10,000 and 20,000 albums per year). *Note:* If you use $t = 3$ and $t = 6$ and work in "months" you'll need to convert 10,000 and 20,000 albums per **year** to $10,000/12$ and $20,000/12$ albums per **month**.

Thus we have $10000 = S(0.25) = a(0.25)^2e^{b(0.25)^2}$ and $20000 = S(0.5) = a(0.5)^2e^{b(0.5)^2}$ which Alpha will happily solve for us.

`solve 10000=a (0.25)^2 e^(b (0.25)^2) and 20000=a (0.5)^2 e^(b (0.5)^2)` ✓

[*Weird Behavior:* If we enter "...$a(0.25)^2$...", Alpha will think "$a(0.25)$" is function instead of the variable "a" times "$(0.25)^2$".]

Alpha finds a real solution $a \approx 201587$ and $b \approx -3.69678$.

$S(t) =$ $\underline{201587t^2e^{-3.69678t^2}}$

The total number of albums sold (according to our model) is $\int_0^\infty S(t)\,dt \approx 12,567$ albums.

`int 201587t^2 e^(-3.69678t^2) from 0 to infinity` ✓

To compute out how long it will take to sell all but one album we need to solve $\int_T^\infty S(t)\,dt = 1$ for the unknown time T. Alpha has trouble doing this in a single step, so we'll break it up into 2 pieces. First, we compute the integral. Then, copy/paste the result of integration and solve the equation.

`int 201587t^2 e^(-3.69678t^2) from T to infinity` ✓

```
 (5039675000 (50 sqrt(924195 pi) erfc(1/100 sqrt(184839/5) T)+
                                184839 e^(-(184839 T^2)/50000) T))/34165455921 = 1 ✓
```

Alpha finds the solution $T = 1.70863...$ which is 1 year and $(0.70863...)(12) = 8.5036...$ months.

After __1__ year and __9__ months only 1 album will remain to be sold.

Alternative Solution: We could work this entire problem in terms of **months** instead of **years**. To do this we would need to convert the 10,000 and 20,000 albums per year figures into $10,000/12$ and $20,000/12$ albums per month. In this case we have: $S(3) = 10000/12$ and $S(6) = 20000/12$ and so...

`solve 10000/12 = a 3^2 e^(b 3^2) and 20000/12 = a 6^2 e^(b 6^2)` ✓

We then find that $a \approx 116.659$ and $b \approx -0.0256721$. Therefore,

$S(t) = 116.659t^2e^{-0.0256721t^2}$

`int 116.659t^2 e^(-0.0256721t^2) from 0 t` ✓

This gives us the same result (as before) of $12,567.3$.

`int 116.659t^2 e^(-0.0256721t^2) from T to infinity` ✓

copy/paste result and solve equation...

```
12567.3-(12567.3 sqrt(T^2) erf(0.160225 sqrt(T^2)))/T+2272.1
                                                      e^(-0.0256721 T^2) T = 1 ✓
```

We find that $T \approx 20.50364$ months. So it takes about 1 year and 9 months to sell all but the final album.

4. After surveying the student body, we have found that the average height of an AppState dorm student is 70 inches. In addition we have found that these heights have a standard deviation of 5. Assume heights are normally distributed.

   What percentage of the student body is between 5 and 6 feet tall? <u>6.327%</u>.

   We are given $\mu = 70$ and $\sigma = 5$. After plugging these values into the normal distribution's density function we need to compute the probability that X is between 60 and 72 inches (5 and 6 feet).

   `int e^(-(x-70)^2/(2 5^2))/(sqrt(2 pi) 5) from 60 to 72` ✓

   We wish to construct a beds which accomodate all but the tallest 2% of students, how long should we make our beds?

   <u>80.269</u> inches.

   We need to solve the equation $\int_X^\infty n(x)\,dx = 0.02$. We'll do this in 2 steps. First, compute the integral. Then copy/paste the result and solve the equation.

   `int e^(-(x-70)^2/(2 5^2))/(sqrt(2 pi) 5) from X to infinity` ✓

   `1/2 erfc((X-70)/(5 sqrt(2))) = 0.02` ✓

   Which gives an answer $X \approx 80.2687$.