Name: Answer Key **Math 1030 Quiz #5** (August 1st, 2011)

1. Most IQ tests are set up to have an average of 100 and a standard deviation of 15.

   We are given that $\mu = 100$ and $\sigma = 15$. Thus the our probability density function is

$$n(x) = \frac{1}{\sqrt{2\pi} \cdot 15} e^{-\frac{(x-100)^2}{2 \cdot 15^2}}$$

   In Alpha, we type "normal distribution" and then copy/paste the density function and then replace "mu" and "sigma" with 100 and 15.

   Mensa requires an IQ of 132 or greater to join. What percentage of the population can join Mensa?

   This is asking us to compute the probability that $X > 132$, so we should use the bounds 132 and ∞ in our integral.

   `int e^(-(x-100)^2/(2 15^2))/(sqrt(2 pi) 15) from 132 to infinity` ✓

$$\int_{132}^{\infty} \frac{1}{\sqrt{2\pi} \cdot 15} e^{-\frac{(x-100)^2}{2 \cdot 15^2}} \, dx \approx 0.0164487$$

   1.645% of the population has an IQ of 132 or higher (and thus is qualified to join Mensa).

   If we wanted to start up a society which accepted the top 5% of all IQ scorers, what would our cut-off score need to be?

   This question is asking us to find a bound on an integral which is equal to 0.05. In particular we need to solve the equation (for c):

$$\int_{c}^{\infty} \frac{1}{\sqrt{2\pi} \cdot 15} e^{-\frac{(x-100)^2}{2 \cdot 15^2}} \, dx = 0.05$$

   To do this we'll need to help Alpha a little. First, we'll make Alpha compute the integral. Then we'll have it solve the equation.

   `int e^(-(x-100)^2/(2 15^2))/(sqrt(2 pi) 15) from c to infinity` ✓

   Copy and paste the expression "1/2 erfc(...)" (this is the value of the integral). We then set this equal to 0.05 and solve.

   `1/2 erfc((c-100)/(15 sqrt(2))) = 0.05` ✓

   Alpha solves and finds that $c \approx 124.673$.

   If we only allow people whose IQ scores are 125 and above to join our society, then only the top 5% of IQ scorers will be admitted to our club.

2. We have the following supply and demand functions:

$$p_s(x) = 50\ln(x+1) + 30 \qquad \text{and} \qquad p_d(x) = -0.2x^3 + 5.8x^2 - 61x + 250.$$

`50ln(x+1)+30 = -0.2x^3+5.8x^2-61x+250` ✓

So that $q_E \approx 3.343$.

`50ln(x+1)+30 where x=3.343` ✓

So $p_E \approx \$103.43$. Therefore,

Market equilibrium $(q_E, p_E) = (3.343, \$103.43)$

$$\begin{aligned} \text{Producer Surplus} &= q_E \cdot p_E - \int_0^{q_E} (\text{supply function})\, dx \\ &= (3.343)(103.43) - \int_0^{3.343} 50\ln(x+1) + 30\, dx \end{aligned}$$

`3.343(103.43) - int 50ln(x+1)+30 from 0 to 3.343` ✓

The producer surplus is \$93.73.