Math 1030 (Section 102) **Final Exam — Part 1** **June 25^{th}, 2010**

Name: Answer Key (PARTIAL) **Be sure to show your work!**

1. (8 points) Find the derivative. Don't worry about simplifying your answers.

(a) $f(x) = \dfrac{e^x + 5x + 1}{\sqrt{x} - 4x^3} + \ln\left(x^5 e^{2x}\right)$

(b) $f(x) = e^{3x} + (2x+1)^{10}\ln(x)$.

2. (12 points) Integration. **Please simplify answers.**

(a) $\displaystyle\int 12x^3 - 5\sqrt{x} + \frac{1}{x^3} + 7\,dx$

(b) $\displaystyle\int e^{-4x} + \frac{2x+1}{x^2+x+3}\,dx$

(c) Suppose that $g'(x) = (5x-10)^{100}$ and $g(2) = 123$. Find $g(x)$.

Math 1030 (Section 102) # Final Exam — Part 2 **June 25^{th}, 2010**

Name: Answer Key (PARTIAL) **Be sure to show your work!**

3. (10 points) Candy owns a paddle boat rental shop on the beach. She has collected the following demand data (q is the number of rentals and p is the price of renting a paddle boat for the afternoon):

$q =$	1	5	25	50
$p =$	\$40	\$35	\$20	\$10

In addition, Cindy's daily fixed costs are \$300 and variable costs are \$5 per boat.

(a) Find an exponential model for Cindy's demand price data.

The exponential trendline is $p(q) =$ ______________________________.

(b) Cindy's daily cost function is $C(q) =$ ______________________________.

(c) Cindy has __________ break even points. They are located at $q =$ ____________________.

4. (10 points) The UN has collected the following data about Billodonia:

$x =$	**0**	0.2	0.6	0.8	**1**
$L(x) =$	**0**	0.05	0.4	0.7	**1**

⟸ We added 0's and 1's to help ensure we get a Lorentz curve.

Where $L(x) = y$ mean that the poorest x percent of Billodonia receive y percent of the income.

(a) Model the Lorenz curve for Billodonia using a **quadratic** function.
Hint: Don't forget to include values for 0 and 1, and remember to check the "Set Intecept" box. The set intercept checkbox appears above the show equation checkbox in the add trendline pop-up

$L(x) =$ $0.8461x^2 + 0.1672x$

(b) Use your model for $L(x)$ to predict how much income the poorest half of the country receives.

$L(0.5) =$ $0.8461(0.5)^2 + 0.1672(0.5) \approx 29.513\%$ (3 decimals please).

(c) The Gini index of Billodonia is 0.268733 (3 decimals please).

`1 - 2 int 0.8461x^2 + 0.1672x from 0 to 1` ✓

5. (10 points) Warren and Wendy just bought a house. They took out a \$150,000 mortgage. Their bank gave them a 30 year mortgage with a 5% (compounded monthly) interest rate.

(a) Their mortgage payment is \$ ____________________ a month.

(b) Fill out the 180$^{\text{th}}$ line of their amortization table and write out the formulas you used in the next line:

Month	Beginning Balance	Payment	Interest	Amount to Principle	End Balance
180					
Formulas:					

(c) How long will it take Warren and Wendy to pay off their house if they increase their payment to \$1,300 a month?

______________ years and ______________ months.

6. (10 points) Joy's store sells 2000 printers each year. When Joy places an order of less than 300 printers, she pays \$50 for each printer. It costs her \$250 to place such an order and her inventory costs are \$5 per printer per year (based on average inventory with all the standard assumptions). On the other hand, when Joy places an order of 300 or more printers, she pays \$40 per printer. However, it costs her \$500 to place a large order and her inventory costs increase to \$10 per printer per year.

(a) Joy's annual cost function is $C(x) = \begin{cases} 2000(50) + 250\left(\frac{2000}{x}\right) + 5\left(\frac{x}{2}\right) & x < 300 \\ 2000(40) + 500\left(\frac{2000}{x}\right) + 10\left(\frac{x}{2}\right) & x \geq 300 \end{cases}$

(b) List **all** of the locations of critical points of $C(x)$ — including negative x's.

```
plot piecewise[{{2000(50)+250(2000/x)+5(x/2),x<300},
                    {2000(40)+500(2000/x)+10(x/2),x>=300}}] on [1,2000] ✓
```

```
derivative 2000(50)+250(2000/x)+5(x/2) ✓
```

We find critical points at $x = 0$ (it's undefined), $x = \pm 447.214$. However, $x = 447.214$ should be thrown away since it is our of bounds for this formula.

```
derivative 2000(40)+500(2000/x)+10(x/2) ✓
```

We find the same critical points (because the author of the exam wasn't paying attention to the problem he wrote). This time 0 and -447.214 are out of bounds (they're less than 300), so $x = 447.214$ is the only critical point coming from this formula.

The critical points of $C(x)$ are located at $x =$ -447.214, 0, 447.214.

The graph of the piecewise function does not even show the first formula's piece (the cost is so much higher than after the discount). So we should look for the minimum after the discount. From the graph we can see the minimum occurs at the critical point $x = 447.214$. Plugging this in will give the optimal (minimum) annual cost (Alpha gives us the answer of 84,472.1).

```
piecewise[{{2000(50)+250(2000/x)+5(x/2),x<300},
                {2000(40)+500(2000/x)+10(x/2),x>=300}}] where x=447.214 ✓
```

(c) Joy's ideal EOQ is 447.214 (3 decimals please).

Her ideal minimal annual cost is $C(x) =$ \$ 84,472.10.

7. (10 points) Given the following demand and supply functions:

$$p_d(q) = 300e^{-0.05q} - 0.2q \qquad \text{and} \qquad p_s(q) = 10\sqrt{q} + 5$$

(a) Find the (exact) area under the supply curve: $p = 10\sqrt{q} + 5$ for $2 \leq q \leq 5$.

Area = $\int_2^5 10\sqrt{q} + 5\, dq \approx 20.785$ (3 decimals please).

```
int 10 sqrt(q) + 5 from 2 to 3 ✓
```

(b) The market equilbrium is $(q_E, p_E) =$ (30.263, \$60.01) (3 decimals please).

```
300e^(-0.05q) - 0.2q = 10 sqrt(q) + 5 ✓
```

```
10 sqrt(q) + 5 where q=30.263 ✓
```

(c) Find the optimal producer surplus. The producer surplus is \$554.89.

```
30.263(60.01) - (int 10 sqrt(q) + 5  from 0 to 30.263) ✓
```

8. (10 points) Elasticity

(a) If $p(q) = -2\ln(q) + 10$ is our demand function and $q = 2$, then point elasticity is

$\varepsilon =$ ________________ (3 decimals please). This is Elastic / Inelastic / Unitary.

Show your work:

(b) Suppose we know that $\varepsilon = 0.75$ at some quantity and this quantity is decreased by 3%.

Then the price p will Increase / Decrease ____________% (percent).

Also, the revenue will Increase / Decrease ____________% (percent).

Show your work:

9. (10 points) Fred's Bowl-a-rama rents a lot of shoes. In fact, Fred rents about 500 pairs of shoes each day. Fred has noticed that the shoe sizes that he rents are normally distributed with a mean size of 8 and standard deviation of 2. *Note:* Shoes sizes go up by increments of 1/2.

(a) How many pairs of shoes sized 12 and larger does Fred rent each day? on average 16 pairs of shoes.

$\mu = 8$ and $\sigma = 2$. Get a copy of the normal distribution's density function and replace mu and sigma with these values. Next, recall that with **discrete** problems we use intervals to represent particular values. Since shoe sizes go up by 1/2's we should split the differences and wrap each size going 1/4 above and below. Thus size 12 is represented by the interval 11.75 up to 12.25. Therefore, sizes 12 and larger are represented by the interval 11.75 up to ∞.

```
int e^(-(x-8)^2/(2 2^2))/(sqrt(2 pi) 2) from 11.75 to infinity ✓
```

We find that $\int_{11.75}^{\infty} n(x)\,dx \approx 0.0303964$ and so there are about $0.0303964 \times 500 = 16.1982$ customers per day which wear size 12 or larger.

(b) Rentals of size 6 and 1/2 and smaller account for 25% of Fred's business.

We need to solve $\int_{-\infty}^{S} n(x)\,dx = 0.25$ for S.

```
int e^(-(x-8)^2/(2 2^2))/(sqrt(2 pi) 2) from -infinity to S ✓
```

Copy/paste result and set equal to 0.25.

```
1/2 (erf((S-8)/(2 sqrt(2)))+1) = 0.25 ✓
```

We get $S \approx 6.65102$. This lies in the range $6.25 \leq S \leq 6.75$, so we should include up to size 6 1/2.

10. (10 points) English Petrol has a broken oil well valve which is spilling oil into a nearby body of water. It is leaking at a rate of $S(t) = te^{-0.02t^2}$ million gallons of oil per day where t is the number of days since the spill began.

(a) When will oil be leaking out at the fastest rate? __5__ days (after the beginning of the spill).

`plot t e^(-0.02t^2) on [0,24]` ✓

The rate of the spill seem to peak close to $t = 5$. Let's compute the derivative of $S(t)$ and look for a critical point close to $t = 5$.

`derivative t e^(-0.02t^2)` ✓

It turns out that $t = 5$ **is** a critical point. This is the location of the maximum rate. [Technically $t = 5$ is the end of day 4 and the beginning of day 5.]

(b) How much oil will leak out during the sixth day of the spill? __2.99446__ million gallons.

The function $S(t)$ gives the rate of the spill (t days after it started). It units are "millions of barrels of oil per day" so to find "millions of barrels of oil" (an amount of oil spilled), we need to multiply by "days". In other words we need to find an area under the curve $y = S(t)$.

Next, keep in mind that $0 \leq t \leq 1$ is day 1, $1 \leq t \leq 2$ is day 2, etc. So day 6 corresponds to the interval $5 \leq t \leq 6$. We need to compute $\int_5^6 S(t)\, dt$ to find how much oil spilled on that day.

`int t e^(-0.02t^2) from 5 to 6` ✓

This integral's value is 2.99446 (millions of gallons of oil).

(c) How long will it take the first 4 million gallons of oil to spill out? __about 3__ days.

We need to find a time X such that from the beginning of the spill (time $t = 0$) until time $t = X$ we spill out 4 million gallons of oil. This means we need to solve the equation $\int_0^X S(t)\, dt = 4$ (remember that integrals are computing *millions* of gallons spilled). Alpha can solve this equation in 2 steps: First, compute the integral. Second, set the result equal to 4 and solve.

`int t e^(-0.02t^2) from 0 to X` ✓

`25.-25. e^(-0.02 X^2) = 4` ✓

$X \approx 2.95257$ days.