

You may **skip ONE** of the following problems.

1. Jane runs the best BBQ restaurant in town. But this requires a lot of napkins. Jane needs about 250 cases of napkins each year. She can purchase a case for \$50. It costs her \$2 per case per year to store them (base inventory costs on average inventory making all of the standard assumptions). Also, it costs \$10 to place an order from her napkin supplier. Let $C(x)$ be the annual cost function.

$$C(x) = \frac{250(50) + 10\left(\frac{250}{x}\right) + 2\left(\frac{x}{2}\right)}{}$$

List **ALL** of the critical points of $C(x)$ including “irrelevant” critical points (points outside the domain of reasonable x values). Round each to 3 decimal places.

plot $250(50)+10(250/x)+2(x/2)$ on $[1,250]$ ✓

derivative $250(50)+10(250/x)+2(x/2)$ ✓

Notice that the derivative has roots ± 50 and don't forget that 0 is also a critical point (the derivative does not exist at $x = 0$).

Critical points: $x =$ -50, 0, and 50

From our original plot, it is obvious that the minimum occurs when $x = 50$ (the only relevant critical point).

$250(50)+10(250/x)+2(x/2)$ where $x=50$ ✓

Jane's **ideal** EOQ is $x =$ 50 and minimum annual cost is $C(x) =$ \$ 12,600.

2. Let $f(x) = \begin{cases} -2x^2 + 3x + 7 & x < 3 \\ x^2 + x - 10 & x \geq 3 \end{cases}$ Sketch the graph of $y = f(x)$ where $-3 \leq x \leq 6$.

plot piecewise[{{-2x^2+3x+7,x<3},{x^2+x-10,x>=3}}] on $[-3,6]$ ✓

derivative $-2x^2+3x+7$ ✓

This derivative has a root at $x = 0.75$. Since $0.75 < 3$ (where this formula is used), $x = 0.75$ is a critical point for our piecewise defined function.

derivative x^2+x-10 ✓

This derivative has a root at $x = -0.5$. However, -0.5 is out of bounds for this formula, so it is not a critical point for our piecewise defined function.

In addition to our critical point at $x = 0.75$, we also have a critical point at $x = 3$ since $f'(3)$ does not exist ($f(x)$ is not continuous at $x = 3$).

$f(x)$ has 2 critical points. They are located at $x =$ 0.75 and 3.
[List **all** critical points. Round to 3 decimal places.]

Example of a piecewise function In ALPHA: The absolute value function can be defined piecewise as

$$\text{piecewise}[\{\{x, x \geq 0\}, \{-x, x < 0\}\}]$$

3. When Jim charges \$2 per drink he usually sells 25 drinks in a day. On the other hand, if Jim charges \$1.50 he usually sells 65 drinks in a day.

Given this data, Elasticity $E = -\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = -\frac{\frac{65-25}{\frac{25+65}{2}}}{\frac{1.50-2}{\frac{2+1.50}{2}}} = -\frac{\frac{40}{45}}{\frac{-0.50}{1.75}} \approx 3.111$.

[$E = 3.111 > 1$ so this situation is elastic.]

If Jim's point elasticity is " $\varepsilon = 1.327$ " when he charges \$1.75, should Jim raise or lower his price to increase his revenue? Or has Jim already maximized his revenue? [Circle the correct answer.]

Raise Prices / Lower Prices / Has Maximized Revenue

Since $\varepsilon > 1$ the percent change in quantity is larger than the percent change in price. So revenue moves in the same direction as quantity. Thus revenue moves in direction opposite to that of the price. Therefore, lower prices mean higher revenues.