Name: ANSWER KEY **Math 1030 Quiz #5** (July 31st, 2013)

You may **skip ONE** of the following problems.

1. We have the following supply and demand functions:

$$p_s(x) = 30\ln(x+1) + 20 \qquad \text{and} \qquad p_d(x) = -0.1x^3 + 0.5x^2 - 2x + 150.$$

To find the Market equilibrium we set the supply and demand curves equal to each other and then plug the equilibrium quantity into either function.

`30ln(x+1)+20=-0.1x^3+0.5x^2-2x+150` ✓ $\Longrightarrow$ $q_E = 9.493$.

`30ln(x+1)+20 where x=9.493` ✓ $\Longrightarrow$ $p_E = p_s(9.493) = p_d(9.493) = \$90.52$

Market equilibrium $(q_E, p_E) =$ (9.493, \$90.52)

Recall that consumer surplus is given by $\displaystyle\int_0^{q_E} p_d(q)\,dq \;-\; q_E \cdot p_E$

`(int -0.1x^3+0.5x^2-2x+150 from 0 to 9.493) - 9.493(90.52)` ✓

The consumer surplus is \$ 414.08

2. Southern Freedonia has determined that their Lorentz curve is given by $L(x) = \dfrac{e^{x^2}-1}{e-1}$.

The first question asks us to solve: $L(x) = 0.10$.

`(e^(x^2)-1)/(e-1) = 0.10` ✓ $\Longrightarrow$ $x = 0.39820$.

The poorest 39.820% of the population receive 10% of the income.

Recall that the Gini Index is given by $1 - 2\int_0^1 L(x)\,dx$.

`1-2 int (e^(x^2)-1)/(e-1) from 0 to 1` ✓

Southern Freedonia's Gini Index is 0.461495 .

3. We know that $P'(q) = -4q^3 + 30q^2 + 50$ for some profit function $P(q)$. In addition, we also know that $q = 10$ is a break even quantity for $P(q)$.

Since we have $P'(q)$, we can find the $P(q)$ up to an arbitrary constant using: $P(q) = \int P'(q)\,dq$.

`int -4q^3+30q^2+50` ✓ $\Longrightarrow$ $P(q) = -q^4 + 10q^3 + 50q + C$

Next, we know that there is a break even quantity at $q = 10$. This means that $P(10) = 0$. If we plug $q = 10$ into $P(q)$, we should get zero. This allows us to find the value of C.

`-q^4+10q^3+50q+C where q=10` ✓ $\Longrightarrow$ $0 = P(10) = C + 500$. Therefore, $C = -500$.

$P(q) =$ $-q^4 + 10q^3 + 50q - 500$