Math 1030 Workshop #5 (Due: July 26th, 2013) **Name:** Answer Key

Please turn in a paper copy and **SHOW YOUR WORK!**

1. Consider the function $f(x) = \begin{cases} 3x^3 + 10x^2 + 5x + 1 & x \leq 0 \\ (8x^4 - 7x)e^{-x^2} + 4 & 0 < x \leq 2 \\ x^2 - 5x + \ln(x) & x > 2 \end{cases}$

Be careful! Wolfram Alpha has a hard time interpreting commands applied to this function. You may want to deal with the function one piece at a time.

$f(x)$ defined in Alpha:

`Piecewise[{{3x^3+10x^2+5x+1,x<=0},{(8x^4-7x)e^(-x^2)+4,0<x<=2},{x^2-5x+ln(x),x>2}}]` ✓

We are (eventually) asked to find the minimum and maximum when $-3 \leq x \leq 5$, so let's plot this function on that interval:

`plot Piecewise[{{3x^3+10x^2+5x+1,x<=0},{(8x^4-7x)e^(-x^2)+4,0<x<=2},{x^2-5x+ln(x),x>2}}] where -3 < x < 5` ✓

We need to find critical points. These are points where the derivative is undefined or equal to zero. First of all, the derivative cannot be defined at $x = 0$ and $x = 2$ since $f(x)$ is discontinuous at these points (see the graph) – this isn't surprising since we usually have a critical point anywhere our piecewise defined function switches formulas.

Next, we would like to find the derivative, but Alpha won't differentiate piecewise defined functions (at least not easily). So we will deal with our function one piece at a time.

`derivative 3x^3+10x^2+5x+1` ✓

We find that this derivative has roots at $x = -1.9351$ and $x = -0.28709$ (after clicking on "Approximate forms" to get decimal approximations). Since both of these point occur when $x < 0$, they lie in the domain of definition for the first formula. So both $x = -1.9351$ and $x = -0.28709$ are a critical points of $f(x)$.

`derivative (8x^4-7x)e^(-x^2)+4` ✓

We find that this derivative has roots at $x = -1.2344$, 0.5, and 1.5637. However, this formula is only valid for $0 < x \leq 2$, so we must discard $x = -1.2344$. So we get 2 more critical points: $x = 0.5$ and $x = 1.5637$.

`derivative x^2-5x+ln(x)` ✓

We find that this derivative has 2 roots: $x = 0.21922$ and $x = 2.2808$. However, $x = 0.21922 \leq 2$ and so this point is outside the domain where the third formula is used (i.e. $x > 2$). On the other hand, $x = 2.2808 > 2$ so this gives us yet another critical point.

(a) Find all of the critical points of $f(x)$. $x =$ -1.9351, -0.28709, 0, 0.5, 1.5637, 2, and 2.2808

(b) Restricting our attention to the interval $[-3, 5]$...

When we look at the graph is it obvious that the minimum occurs at the extreme end point $x = -3$ or at the critical point $x = 2.2808$ (just after $x = 2$). The maximum looks to occur either at the peak at $x = -1.9351$ or at the peak at $x = 1.5637$. By plugging in these values, we could find our min and max.

Alternatively, we could plug in all of the critical points along with the end points $x = -3$ and $x = 5$ and see which values are the biggest and smallest — this is essentially what we'd need to do if we didn't have a reliable graph. [*Note:* Technically, we would also need to check right & left hand limits at each point of discontinuity. The extreme value theorem doesn't apply directly since our function isn't continuous. Typically, we won't worry about these technicalities and just rely on the graph.] I will plug in all critical points, end points, and take "limits" where formulas switch (by "limits" I mean plug in ends of intervals of definition)...

`3x^3+10x^2+5x+1 at x=[-3,-1.9351,-0.28709,0]` ✓

This gives us $f(-3) = -5$, $f(-1.9351) = 7.03202$, $f(-0.28709) = 0.31777$, and $f(0) = 1$.

`(8x^4-7x)e^(-x^2)+4 at x=[0,0.5,1.5637,2]` ✓

This gives us $\lim_{x \to 0^+} f(x) = 4$, $f(0.5) = 1.6636$, $f(1.5637) = 7.19834$, and $f(2) = 6.08798$.

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x^2-5x+ln(x) at x=[2,2.2808,5] ✓
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This gives us $\lim_{x\to 2^+} f(x) = -5.30685$, $f(2.2808) = -5.37743$, and $f(5) = 1.60944$.

The maximum value of $f(x)$ is <u>7.19834</u>. This occurs when $x =$ <u>1.5637</u>.

The minimum value of $f(x)$ is <u>-5.37743</u>. This occurs when $x =$ <u>2.2808</u>.

2. Todd's Rentals (located in Todd, NC) rents heavy duty pressure washers. Each washer costs \$800. Over time Todd has found that a typical pressure washer will require \$50 of repairs during its first year of operation, \$140 of repairs during its second year of operation, and \$305 of repairs during its third year of opertion.

   Use Excel to find a power model for the **average** annual repair costs. Then model the average annual cost of operating a pressure washer using a function of the form: $A(t) = \dfrac{C}{t} + Rt^r$ where C is the cost of purchasing the TV and Rt^r models the repair costs.

   Notice that the average repair cost for year 1 is \$50 (given). The average repair cost for the first 2 years is $\dfrac{\$50 + \$140}{2} =$ \$95. Finally, the average repair cost for the first 3 years is $\dfrac{\$50 + \$140 + \$305}{3} = \$165$. So we should make a table in Excel with the following data:

Years Owned	1	2	3
Average Repair Cost	\$50	\$95	\$165

   If we highlight this data, create a scatter plot, and add a "power" trendline. We find that $y = 48.678x^{1.0694}$ best fits this data.

$$A(t) = \frac{800}{t} + 48.678t^{1.0694}$$

   We need to find the minimum value of $A(t)$. So we take it's derivative and find critical points.

```
derivative 800/t + 48.678t^1.0694 ✓
```

   We can see that $A'(t)$ is not defined at $t = 0$ (but we should only consider $t > 0$). We have a root at $t = 3.74465$. Looking at the plot where $1 \le t \le 6$ we see...

```
plot 800/t + 48.678t^1.0694 on [1,6] ✓
```

   When $t =$ <u>$t = 3.74465$</u>, $A(t)$ is minimized. [Keep 5 decimal places.]

   Let's plug this value into $A(t)$...

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800/t + 48.678t^1.0694 at t=3.74465 ✓
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   We find that $C(3.74465) = \$413.41$. Also, $t = 3.74465$ is 3 years and about 9 months.

   Todd should replace his pressure washers every <u>3</u> years and <u>9</u> months.
   [Round up to the next whole month.]

   If he does this, his average annual cost (per washer) should be \$ <u>413.41</u>.

3. Stacy runs a small fruit stand just outside of Boone. She pays \$1.25 per carton of blueberries and has found that her average storage costs are \$0.25 per carton per year (base inventory costs on average inventory making all of the standard assumptions). Suppose Stacy is charged \$20 every time she places an order. Let $C(x)$ be Stacy's annual cost function.

   In general we have,

$$\begin{aligned}
\text{annual cost} &= \text{Base Cost} + \text{Shipping Cost} + \text{Storage Cost} \\
&= (\text{\# of items})(\text{price per item}) + (\text{shipping per order})(\text{\# of orders}) + (\text{ave. inv.})(\text{inv. per item per yr.}) \\
&= (\text{\# of items})(\text{price per item}) + (\text{shipping per order})\left(\frac{\text{\# of items}}{x}\right) + \left(\frac{x}{2}\right)(\text{inv. per item per yr.})
\end{aligned}$$

(a) If Stacy sells 1200 cartons each year, $C(x) = 1.25(1200) + 20\left(\frac{1200}{x}\right) + 0.25\left(\frac{x}{2}\right)$

Let's plot the cost function on the interval: $1 \leq x \leq 1200$ and compute its derivative to find critical points.

`plot 1.25(1200)+20(1200/x)+0.25(x/2) on [1,1200]` ✓

`derivative 1.25(1200)+20(1200/x)+0.25(x/2)` ✓

The only critical point of relevance is $x = 438.178$.

`1.25(1200)+20(1200/x)+0.25(x/2) at x=438.178` ✓

Her **ideal** EOQ is 438.178 and her **ideal** minimum annual cost is $1,609.54.

(b) If Stacy sells 150 cartons each year, $C(x) = 1.25(150) + 20\left(\frac{150}{x}\right) + 0.25\left(\frac{x}{2}\right)$

Let's plot the cost function on the interval: $1 \leq x \leq 150$ and compute its derivative to find critical points.

`plot 1.25(150)+20(150/x)+0.25(x/2) on [1,150]` ✓

`derivative 1.25(150)+20(150/x)+0.25(x/2)` ✓

This time there are no critical points of relevance. They all occur outside our domain: $1 \leq x \leq 150$. So the minimum must occur at an endpoint. Obviously $x = 150$ gives the minimum (not $x = 1$).

`1.25(150)+20(150/x)+0.25(x/2) at x=150` ✓

Her **ideal** EOQ is 150 and her **ideal** minimum annual cost is $226.25.

(c) Suppose that Stacy sells 2000 cartons per year and gets a small discount if she places a large order. For orders of 1500 or more blueberry cartons, she pays $1.12 each. However, her shipping costs double to $40 for a large shipment and her inventory costs double to $0.50 per carton as well.

$$C(x) = \begin{cases} 1.25(2000) + 20\left(\frac{2000}{x}\right) + 0.25\left(\frac{x}{2}\right) & x < 1500 \\ 1.12(2000) + 40\left(\frac{2000}{x}\right) + 0.5\left(\frac{x}{2}\right) & x \geq 1500 \end{cases}$$

```
plot Piecewise[{{1.25(2000)+20(2000/x)+0.25(x/2),x<1500},
                {1.12(2000)+40(2000/x)+0.5(x/2),x>=1500}}] on [1,2000] ✓
```

From the plot it is apparent that the minimum occurs when $x < 1500$, so we can discard the second formula and just focus on the first one. We need to find the first formula's critical points.

`derivative 1.25(2000)+20(2000/x)+0.25(x/2)` ✓

The only critical point of relevance is $x = 565.685$. From the original graph, we can tell that this is indeed our ideal EOQ.

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Piecewise[{{1.25(2000)+20(2000/x)+0.25(x/2),x<1500},
           {1.12(2000)+40(2000/x)+0.5(x/2),x>=1500}}] at x=565.685 ✓
```

Cindy's **ideal** EOQ is 565.685. Her **ideal** minimum annual cost is $2,641.42.