Math 1030 Workshop #6 **Name:** Answer Key

DUE: Wednesday, July 31st Please turn in a paper copy and **SHOW YOUR WORK!**

1. Jessica is selling low end student violins at her music store. Over time she collected the following demand data:

   (a) Compute elasticity (round to 3 decimals places):

Violins Sold	5	20	150	180
Price	\$200	\$175	\$100	\$80
Elasticity	9	2.804	0.818	×

I computed elasticity in Excel using something like: `=-((B1-A1)*(A2+B2))/((B2-A2)*(A1+B1))` where `A1` and `B1` hold quantities and `A2` and `B2` hold prices.

   (b) Model this demand price (in Excel) using an **exponential** trendline.

   $p_d(q) =$ $199.96e^{-0.005q}$

   According to this model, we will sell $q = 102.125 \approx 102$ violins if we set our price at \$120.

   We get the quantity corresponding to \$120 by solving: `199.96e^(-0.005q)=120` ✓

   When $p_d(q) = \$120$, our point elasticity is $\varepsilon =$ 1.958 (round to 3 decimal places).

   Compute $\varepsilon = -\dfrac{p/q}{dp/dq} = \dfrac{200}{q}$: `-(199.96e^(-0.005q)/q)/(derivative 199.96e^(-0.005q))` ✓

   Then plug-in $q = 102.125$: `200/q where q=102.125` ✓

   Circle the correct answer: We are currently charging \$120 for a student violin and want to **increase** revenue, we should raise / **[lower]** our price.

   Our point elasticity at $p = \$120$ is $\varepsilon = 1.958 > 1$ (elastic). Since $\varepsilon > 1$, the top of the fraction (percent change in quantity) dominates. Thus revenue and quantity move in the same direction and so revenue and price move in opposite directions.

   We know that revenue should be maximized when $\varepsilon = 1$. Solving $\varepsilon = \dfrac{200}{q} = 1$ yields $q = 200$.

   To get the corresponding price: `199.96e^(-0.005q) at q=200` ✓

   What quantity and price will **maximize** revenue? $q =$ 200 $p =$ 73.56

2. Warren is building a odd shaped patio shaped like the portion of the xy-plane which is bounded by the x-axis and $y = 7|x| - 3x^2 + 2$. He needs to determine the area of this region so he knows how much concrete he needs to purchase.

   We are looking for roots: `7|x|-3x^2+2` ✓ finds $x = \pm 2.5907$ are the roots.

   `plot 7|x|-3x^2+2 on [-2.5907,2.5907]` ✓

   Find the x-coordinates of the points where $y = 7|x| - 3x^2 + 2$ crosses the x-axis. [Round to 4 decimal places.]

   $x =$ -2.5907 and $x =$ 2.5907

   Determine the area of this region 3 different ways: (1) Using a right hand rule approximation with $n = 10$ rectangles. (2) Using Simpson's rule with $n = 4$ and (3) Compute the area exactly using Alpha. [Round each answer to 4 decimal places.]

   Download the right hand rule and Simpson's rule area approximation Excel sheets. We have $a = -2.5907$, $b = 2.5907$. For the right hand rule $n = 10$ and for Simpson's rule $n = 4$. Our function is entered in Excel as: `=7*ABS(B14)-3*B14^2+2`

   To get the exact area under the curve use a definite integral: $\int_{-2.5907}^{2.5907} 7|x| - 3x^2 + 2\,dx$.

   `int 7|x|-3x^2+2 from -2.5907 to 2.5907` ✓

   Since our curve is just 2 parabolas stuck together and Simpson's rule uses parabolas to estimate area under the curve, Simpson's gave the exact answer!

   Right hand rule: 21.8732 Simpson's rule: 22.5687 Exact area: 22.5687

3. Suppose that $R(t) = 5\ln(t+1) - t + 10$ models the rate of consumption of pancakes at Fun Time Summer Camp. t is the number of minutes from the time that breakfast is served and $R(t)$ is measured in pancakes per minute.

   The campers stop eating pancakes when the rate hits zero. This means we need to find the roots of our rate function. Alpha finds 2 real roots: $t \approx -0.887$ and 26.587.

   `5ln(t+1)-t+10` ✓

   When will the campers stop eating pancakes? after $t =$ <u>26.587 minutes</u>

   Since $R(t)$ is a rate function (pancakes eaten per minute), area under the curve $y = R(t)$ gives us the number of pancakes eaten. So to find pancakes eaten from the beginning of breakfast until the campers were done, we need to integrate from $t = 0$ to 26.587.

   `int 5ln(t+1)-t+10 from 0 to 26.587` ✓

   The total number of pancakes eaten is <u>$237.079 \approx 237$</u> pancakes.

   To find *how long* it takes the first 100 pancakes to get eaten, we must set up an equation involving an integral and solve for a bound. The integral: $\int_0^T 5\ln(t+1) - t + 10\,dt$ computes how many pancakes were eaten in the first T minutes. So setting this equal to 100 and solving for T should give us our answer.

   `(int 5ln(t+1)-t+10 from 0 to T) = 100` ✓

   How long does it take the first 100 pancakes to get eaten? <u>7.475</u> minutes.

4. After surveying the student body, we have found that the average weight of a Western Hoople University (WHU) dorm student is 145 lbs. In addition we have found that these weights have a standard deviation of 15. Assume these weights are normally distributed.

   In Wolfram Alpha we type `normal distribution` ✓ to retrieve the probability density function formula (click on the formula). Then replace μ with 145 and σ with 15.

   To get the percentage of students which weight between 110 and 130 lbs. we need to compute the integral $\int_{110}^{130} n(x)\,dx$ (where $n(x)$ is our normal distribution).

   `int e^(-(x-145)^2/(2 15^2))/(sqrt(2 ) 15) from 110 to 130` ✓

   What percentage of the student body weighs between 110 and 130 lbs? <u>14.884%</u>.

   To get the number of students who weigh less than 100 lbs. we need to compute the probability a student weighs less than 100 lbs.: $\int_{-\infty}^{100} n(x)\,dx$ and then scale this by 1500.

   `1500 int e^(-(x-145)^2/(2 15^2))/(sqrt(2 ) 15) from -infinity to 100` ✓

   If WHU has 1,500 dorm students, how many students weight less than 100 pounds? <u>$2.025 \approx 2$</u> students.

   We are looking for a cut-off, so we're solving for a bound. Specifically we need to solve the equation: $\int_X^{\infty} n(x)\,dx = 2\%$.

   `(int e^(-(x-145)^2/(2 15^2))/(sqrt(2 ) 15) from X to infinity)=0.02` ✓

   How much would you have to weigh to be in the heaviest 2% of the student body? <u>175.806</u> lbs.