

You may **skip ONE** of the following problems.

1. Emily owns “Shoes by Twos”. She sells 2000 pairs of a particular style of shoe each year. She can purchase these shoes for \$20 a pair. Shipping costs \$100 per order. Also, it costs her \$1 to store a pair of shoes for a year (base inventory costs on average inventory with all of the standard assumptions). Let  $C(x)$  be Emily’s annual cost function for these shoes.

$$C(x) = \underline{2000(20) + 100\left(\frac{2000}{x}\right) + 1\left(\frac{x}{2}\right)}$$

List **ALL** of the critical points of  $C(x)$  including “irrelevant” critical points (points outside the domain of reasonable  $x$  values). Round each to 3 decimal places.

derivative  $2000(20) + (2000/x)100 + 1(x/2)$  ✓

We find that the derivative has 2 roots:  $x = \pm 200\sqrt{10} \approx \pm 632.46$ . Of course the function isn’t defined at  $x = 0$ , so we should add this to our list.

Critical points:  $x = \underline{-200\sqrt{10} \approx -632.46, \quad 0, \quad \text{and} \quad 200\sqrt{10} \approx 632.46}$

If we plot our cost function...

plot  $2000(20) + (2000/x)100 + 1(x/2)$  on  $[1, 2000]$  ✓

...we can see that the cost function is minimized at the (only relevant) critical point  $x = 632.46$ . Finally, we should plug this in to find the minimal cost.

$2000(20) + (2000/x)100 + 1(x/2)$  at  $x=632.46$  ✓

Emily’s **ideal** EOQ is  $x = \underline{632.46}$  pairs of shoes and minimum annual cost is  $C(x) = \$ \underline{\$40,632.50}$ .

2. Let  $f(x) = \begin{cases} x^2 + 2x + 7 & x \leq 1 \\ -x^2 - 3x + 15 & x > 1 \end{cases}$  Sketch the graph of  $y = f(x)$  where  $-2 \leq x \leq 3$ .

We can either plot the two parabolas together and then erase the irrelevant parts of each one or use a piecewise command.

plot piecewise[{{x^2+2x+7,x<=1},{-x^2-3x+15,x>1}}] on  $[-2, 3]$  ✓

To find the critical points we should differentiate each piece.

derivative  $x^2+2x+7$  ✓

We find that this derivative has a root at  $x = -1$ . Since this is within the bounds for this formula (i.e.  $x \leq 1$ ), this is a critical point for  $f(x)$ .

derivative  $-x^2-3x+15$  ✓

This derivative has a root at  $x = -1.5$ . Since this is outside the bounds for this formula (i.e.  $x > 1$ ), this isn’t a critical point for  $f(x)$ .

Finally, we get a critical point at  $x = 1$  since our function is discontinuous and so  $f'(1)$  does not exist.

$f(x)$  has 2 critical points. They are located at  $x = \underline{-1}$  and 1.  
[List **all** critical points. Round to 3 decimal places.]

Example of a piecewise function In ALPHA: The absolute value function can be defined piecewise as

piecewise[{{x, x >= 0}, {-x, x < 0}}]

3. Bob typically sells 10 sketches when he charges \$50 per sketch. On the other hand, if Bob charges \$20, he typically sells 22 sketches.

$$- \left( \frac{22-10}{((10+22)/2)} \right) / \left( \frac{20-50}{((50+20)/2)} \right) \checkmark$$

Given this data, Elasticity  $E = \underline{0.875}$ .

[Since  $E < 0.875$ , this situation is inelastic.]

If Bob's point elasticity is " $\varepsilon = 0.755$ " when he charges \$35 per sketch, should Bob raise or lower his price to increase his revenue? Or has Bob already maximized his revenue? [Circle the correct answer.]

Since  $\varepsilon = 0.755 < 1$ , this situation is inelastic. This means that the percent change in price exceeds the percent change in quantity. Thus revenue moves with price. So as prices go up, so does revenue.

Raise Prices      /      Lower Prices      /      Has Maximized Revenue