Name: ANSWER KEY **Math 1030 Quiz #5** (July 30st, 2013)

You may **skip ONE** of the following problems.

1. We have the following supply and demand functions: $p_s(x) = 20e^{x/10}$ and $p_d(x) = -x^2 + 500$.

   Market equilibrium $(q_E, p_E) =$ (19.1025, \$135.09)

   The consumer surplus is \$ 4,647.08 The producer surplus is \$ 1,429.68

   I used the following ALPHA commands...

   We don't really need to plot our functions, but here's a plot of the supply and demand curves...

   `plot {20e^(x/10), -x^2+500} where 0<x<30` ✓

   First, we need to find the market equilibrium. This is where the curves intersect. Setting our functions equal to each other we get...

   `20e^(x/10) = -x^2+500` ✓

   Alpha tells us that the curves intersect at $x = 19.1025$. We need to plug this into either function (they're equal here) to find the equilibrium price.

   `-x^2+500 at x=19.1025` ✓

   Alpha tells us that when $q_E = 19.1025$, $p_E = 135.094$.

   Now we're ready to compute the consumer and producer surpluses.

   $$\text{Consumer Surplus} = \int_0^{q_E} \text{demand}\, dq - q_E \cdot p_E = \int_0^{19.1025} (-q^2 + 500)\, dq - (19.1025)(135.094)$$

   `(int -x^2+500 from 0 to 19.1025) - 19.1025(135.094)` ✓

   Alpha finds that the consumer surplus is \$4,647.08.

   $$\text{Producer Surplus} = q_E \cdot p_E - \int_0^{q_E} \text{supply}\, dq = (19.1025)(135.094) - \int_0^{19.1025} 20e^{q/10}\, dq$$

   `19.1025(135.094) - int 20e^{x/10} from 0 to 19.1025` ✓

   Alpha finds that the producer surplus is \$1,429.68.

2. Whipple County has determined that their Lorentz curve is given by $L(x) = 0.5x^3 + 0.4x^2 + 0.1x$.

   The poorest 54.065% percent of the population receives 25% of the income.

   The poorest 25% percent of the population receives 5.781% of the income.

   Whipple County's Gini Index is 0.3833.

   I used the following ALPHA commands...

   As in the previous problem, we don't really need a graph. But I create one anyway. Here we plot the Lorentz curve together with $y = x$ (i.e. total equality).

   `plot {0.5x^3+0.4x^2+0.1x, x} on [0,1]` ✓

   To find the poorest x percent which receive 25% of the income requires us to solve the equation $L(x) = 0.25$.

   `0.5x^3+0.4x^2+0.1x=0.25` ✓

   Alpha finds that $L(0.540647) = 0.25$. Thus the poorest 54.065% of the population receive 25% of the income.

   To find how much the poorest 25% of the population receives, we simply need to compute $L(0.25)$.

   `0.5x^3+0.4x^2+0.1x at x=0.25` ✓

   Alpha finds that $L(0.25) = 0.0578125$. Thus the poorest 25% of the population receives 5.781% of the income.

   Finally, recall that the Gini Index is $\dfrac{\text{Area between } x \text{ and } L(x)}{\text{Area under } x} = \dfrac{(1/2) - \int_0^1 L(x)\, dx}{1/2} = 1 - 2\int_0^1 L(x)\, dx.$

   `1 - 2 int 0.5x^3+0.4x^2+0.1x from 0 to 1` ✓

   Alpha finds that the Gini Index is 0.383333.

3. We know that $P'(q) = -4q^3 + 300q^2 - 1400q - 2000$ for some profit function $P(q)$. In addition, we also know that $q = 15$ is a break even quantity for $P(q)$.

$P(q) =$ $\underline{\quad -q^4 + 100q^3 - 700q^2 - 2000q - 99375 \quad}$

I used the following ALPHA commands...

We need to find an antiderivative of $P'(q)$. Thus we need to compute the indefinite integral:
$\int P'(q)\,dq = \int(-4q^3 + 300q^2 - 1400q - 2000)\,dq$.

`int -4q^3+300q^2-1400q-2000` ✓

Alpha finds that $\int P'(q)\,dq = -q^4 + 100q^3 - 700q^2 - 2000q + C$. Our next task is to figure out what "C" is.

We are told that this profit function has a break even quantity at $q = 15$. This means that $P(15) = 0$. Let's compute $P(15)$ using our newly found formula.

`-q^4+100 q^3-700 q^2-2000 q+C at q=15` ✓

Alpha tells us that $P(15) = C + 99375$. Since $P(15) = 0$, we must have $C + 99375 = 0$ so that $C = -99375$. Therefore, our profit function is $P(q) = -q^4 + 100q^3 - 700q^2 - 2000q - 99375$.

If we punch this into Alpha, it can confirm that $P(q)$ does have a break even point at $q = 15$ and that the derivative matches.

`-q^4+100 q^3-700 q^2-2000 q-99375` ✓