Math 1030 Workshop #6 **Name:** ANSWER KEY

DUE: Thursday, July 30st Please turn in a paper copy and SHOW YOUR WORK!

1. Apple has rolled out its latest product…the iPen. The local iStore has collected the following iData about their iProduct's demand:

iPens Sold	15	100	200	350
Price	$1,000	$225	$100	$50

(a) Compute elasticity (round to 3 decimals places):

iPens Sold	15	100	200	350
Price	$1,000	$225	$100	$50
Elasticity	1.1683	0.8667	0.8182	× ×

(b) Model this demand price (in Excel) using an **power** trendline: $p_d(q) =$ $13564\,q^{-0.932}$

At this point, I'll switch from Excel to using Wolfram Alpha. Alpha does a better job of solving equations.
To find what quantity goes with $p_d = \$75$ we use… `13564 q^-0.932 = 75` ✓
Alpha solves this equation and tell us that $q \approx 264.256$.

According to this model, the store will sell 264 iPens if they set their price at $75.

Recall that point elasticity is $\varepsilon = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{p}{d} \Big/ \frac{dp}{dq}$. Since we have our demand function: $p_d(q) = 13564q^{-0.932}$ (price in terms of quantity), we need to use the second expression (we will be computing $\frac{dp}{dq}$ not $\frac{dq}{dp}$). Let's use Alpha to find this function, then we can plug in $q = 264.256$ and $p = 75$.

`derivative 13564 q^-0.932` ✓ Alpha finds that the derivative is $-\frac{12641.6}{q^{1.932}}$. Putting this together, we find that $\varepsilon = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{13564\,q^{-0.932}}{q} \Big/ \frac{-12641.6}{q^{1.932}}$. Now plug in $q = 264.256$…

`-(13564q^-0.932 / q)/(-12641.6/q^1.932) at q=264.256` ✓

When $p_d(q) = \$75$, our point elasticity (using this model) is $\varepsilon =$ 1.073 (round to 3 decimal places).

Since $\varepsilon > 1$, quantity is dominant and so revenue moves against price. To increase revenue they should drop the price.

Circle the correct answer: The store is currently charging $75 for an iPen and wants to **increase** its revenue, they should raise / [lower] their price.

To find where revenue is maximized we must solve the equation: $\varepsilon = 1$.

`-(13564q^-0.932 / q)/(-12641.6/q^1.932) = 1` ✓

Alpha correctly returns "False". This happens because our revenue function has **no maximum**.

~~What quantity and price will **maximize** revenue? $q =$ ______ $p =$ ______~~

2. Suppose that some items point elasticity is $\varepsilon = 1.5$ at some price. If the price is lowered by 5%, …

We have that $1.5 = \varepsilon = -\frac{\text{percent change in quantity}}{\text{percent change in price}} = -\frac{\text{percent change in quantity}}{-0.05}$. Thus the percent change in quantity is $1.5 \times 0.05 = 0.075$.

Suppose that revenue used to be $R = pq$. Then if the price is lowered by 5%, it becomes $0.95p$. If the quantity is raised by 7.5%, it becomes $1.075q$. Putting this together, our new revenue is $R = (0.95p)(1.075q) = 1.02125pq$. Thus the new revenue is 102.125% of the old revenue.

The quantity is [raised] by 7.5% and revenue is [raised] by 2.125%.

3. Kyle is painting a very odd wall. Its height varies quite a lot. He needs to estimate the square footage of the wall so he can figure out how much paint to buy. Suppose that the wall is 30 feet long. Let x be the number of feet from the start of the wall and y be the height (in feet) of the wall as measured by Kyle...

$x =$	0	3	6	9	12	15	18	21	24	27	30
$y =$	5	7	8	7	10	9	6	5	4	6	7

Approximate the area of this wall in 2 different ways: (1) Using a right hand rule approximation with $n = 10$ rectangles and (2) Using Simpson's rule with $n = 10$. [Round each answer to 4 decimal places.]

For both approximations we can use our textbook's Excel sheets. In both cases $a = 0$, $b = 30$, $n = 10$ (so $\Delta x = 3$). Once a, b, and n are typed in, the sheets list a bunch of x values (partition points). Notice that Simpson's rule lists $0, 3, \ldots, 30$ while the Righthand rule sheet leaves off 0 (it's the leftmost point, so righthand rule doesn't use it).

We don't have a formula for our $f(x)$, but Kyle provided us with the values of $f(x)$ that we need. Just type in each of the values. So for the righthand rule sheet type in $7, 8, 7, 10$, etc. For Simpson's type in $5, 7, 8, 7$, etc. The sheets then compute our desired answers.

Right hand rule: 207 square feet   Simpson's rule: 204 square feet

4. Suppose that $R(t) = (t + 0.5)^3 e^{-t}$ models the construction rate of micro-houses (in millions of houses built per year). t is the number of years since January 1, 2015.

Our first question asks about the rate function itself. We need to solve $R(t) = 0.01$ (remember the rate is in millions of houses so $1 = 1$ million houses, $0.1 = 100{,}000$ houses, and $0.01 = 10{,}000$ houses).

`(t+0.5)^3e^(-t) = 0.01` ✓

When (after the initial peak) will the construction rate drop to 10,000 houses per year? $t =$ 12.239 years

January 1, 2025 corresponds with $t = 10$. We need to "add up" all of the houses built after $t = 10$. In other words, we need to compute $\int_{10}^{\infty} R(t)\, dt$.

`int (t+0.5)^3e^(-t) from 10 to infinity` ✓

Alpha finds that $\int_{10}^{\infty} R(t)\, dt \approx 0.070705$ (millions of houses).

How many micro-houses will be built after January 1, 2025? 70,705 houses .

We need to find X such that the number of houses built from $t = 0$ to $t = X$ is 5,000,000. In other words, we need to solve $\int_0^X R(t)\, dt = 5$.

`(int (t+0.5)^3e^(-t) from 0 to X) = 5` ✓

Alpha finds that $X \approx 3.20646$ years. Notice that 0.20646 years $= 0.20646 \times 12 = 2.47752$ months. 3 years and 2.468 months from January 1, 2015 is somewhere in the middle of March 2018.

When will the 5,000,000$^{\text{th}}$ micro-house be completed? March 2018 .

5. After surveying the residents of Raccoon City, we have found that the lifespan of an average citizen is 35 years with a standard deviation of 3. Assume that these lifespans are normally distributed.

Type in "normal distribution" to get the required formula. Then set $\mu = 35$ and $\sigma = 3$. Our first question asks us to compute $\int_{20}^{40} n(x)\, dx$.

`int e^(-(x-35)^2/(2 3^2))/(sqrt(2 pi) 3) from 20 to 40` ✓

What percentage of residents live between 20 and 40 years? 95.221%

For the next question we need to know the percentage of residents that live more than 50 years and then multiply by the number of residents.

`10000000 int e^(-(x-35)^2/(2 3^2))/(sqrt(2 pi) 3) from 50 to infinity` ✓

If Raccoon City has 10,000,000 residents, how many live more than 50 years? 2.867 = about 3 people

For the last question we need to solve the equation $\int_{-\infty}^{X} n(x)\, dx = 0.10$.

`(int e^(-(x-35)^2/(2 3^2))/(sqrt(2 pi) 3) from -infinity to X) = 0.10` ✓

What is the cut-off determining the 10% with the shortest lifespans? 31.155 years old