Name: Answer Key **Math 1030 Quiz #4** (July 26th, 2016)

You may **skip ONE** of the following problems.

1. Bob's Plastic Pink Flamingo Shack sells 5,000 flamingos each year. Bob can purchase a pink flamingo for \$2. His supplier has a \$50 shipping fee. Also, Bob has figured out that it costs him \$0.75 to store a flamingo for a year (base inventory costs on average inventory with all of the standard assumptions). Let $C(x)$ be Bob's annual cost function for his flamingos.

   $C(x) = 5000(2) + 50\left(\frac{5000}{x}\right) + 0.75\left(\frac{x}{2}\right)$

   `plot 5000(2)+50(5000/x)+0.75(x/2) on [1,5000]` ✓

   `derivative 5000(2)+50(5000/x)+0.75(x/2)` ✓

   The derivative is 0 at $x = \pm 816.497$. In addition, the derivative is undefined at $x = 0$. These are all of the $C(x)$'s critical points. Of course, -816.497 and 0 are out of bounds critical points, so they should not be considered when looking for a minimum (or maximum).

   List **ALL** of the critical points of $C(x)$ including "irrelevant" critical points (points outside the domain of reasonable x values). Round each to 3 decimal places.

   Critical points: $x =$ -816.497, 0, 816.497

   The minimum and maximum value of a continuous function restricted to a closed interval must occur at either a critical point or an end point. Thus if we check $x = 1$, 816.497, and 5000, we will find both the minimum and maximum values of $C(x)$ restricted to $1 \leq x \leq 5000$. Of course, from the graph we already know that the minimum occurs at $x = 816.497$ (so plugging in all 3 points is kind of silly)...

   `5000(2)+50(5000/x)+0.75(x/2) where x=[1, 816.497, 5000]` ✓

   Bob's **ideal** EOQ is $x =$ 816.497 and minimum annual cost is $C(x) = \$$ 10,612.4 .

   Note: The above Wolfram ALPHA computation also reveals that the maximum occurs when $x = 1$ and results in a maximal cost of $C(1) = \$260,000$ (Of course, ordering the flamingos one at a time is a *very* bad idea).

2. Let $f(x) = \begin{cases} x^3 + 3x^2 + x & x < -1 \\ -x^2 + x + 3 & x \geq -1 \end{cases}$ Sketch the graph of $y = f(x)$ where $-2 \leq x \leq 2$.

   `plot piecewise[{ {x^3+3x^2+x, x<-1}, {-x^2+x+3,x>=-1} }] on [-2,2]` ✓

   Let's find the critical points by differentiating our function one piece at a time (and filtering out irrelevant points).

   `derivative x^3+3x^2+x` ✓

   The derivative of $x^3 + 3x^2 + x$ has roots at $x \approx -1.8165$ and -0.18350. The first point is relevant and the second point is not (it is out of bounds for this formula since it isn't less than -1).

   `derivative -x^2+x+3` ✓

   The derivative of $-x^2 + x + 3$ has a root at $x = 0.5$. This is a relevant critical point since $0.5 \geq -1$ (in bounds for this formula).

   Finally, notice from the graph of our piecewise function that it has a sharp corner at $x = -1$ (where the formulas switch). This means that $x = -1$ is also a critical point.

   $f(x)$ has 3 critical points. They are located at $x =$ -1.817, -1, and 0.5 .
   [List **all** critical points. Round to 3 decimal places.]

3. Wendy typically sells 50 frosties when she charges \$1.50 per frostie. On the other hand, if Wendy charges \$2, she typically sells only 12 frosties.

Given this data, Elasticity E = $-\dfrac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = -\dfrac{\frac{q_2 - q_1}{\frac{q_1+q_2}{2}}}{\frac{p_2 - p_1}{\frac{p_1+p_2}{2}}} = -\dfrac{\frac{12-50}{(12+50)/2}}{\frac{2-1.50}{(2+1.50)/2}} = -\dfrac{-38/31}{0.50/1.75} \approx 4.290$.

If Wendy's point elasticity is "$\varepsilon = 2.25$" and she lowers her price 5%, what should Wendy expect to happen to her revenue? [Circle the correct answer.]

Remember that elasticity is negative percent change in quantity over percent change in price. This means that if $\varepsilon = 2.25 > 1$, then the change in quantity dominates and revenue goes with quantity (and against price). Also, if we drop price, our quantity should go up (price and quantity should typically move in opposite directions). Putting this together, if we drop the price, the quantity should go up and the revenue should go up with the quantity (since our situation is "elastic": $\varepsilon > 1$).

Now for more precise information. We have that $2.25 = \varepsilon = -\%\Delta q/\%\Delta p = -\%\Delta q/(-5\%) = \%\Delta q/0.05$. This means that the percent change in quantity is $\%\Delta q = 2.25(0.05) = 0.1125 = 11.25\%$.

[Revenue Increases] / Revenue Decreases

Wendy should see her quantity sold **[Increase]** / Decrease by 11.25 %.

In fact, we have $p_{new} = 95\% p_{old}$ and $q_{new} = 111.25\% q_{old}$. This means that $R_{new} = p_{new} q_{new} = 0.95 p_{old} 1.1125 q_{old} = 0.95(1.1125) p_{old} q_{old} = 1.056875 R_{old}$. This means that revenue should increase by 5.6875%.