

Name: \_\_\_\_\_

1. Hank is a ticket scalping middleman. He has collected the following data about tickets to a local themepark: [Note: For this whole problem, restrict your attention to the range  $0 \leq q \leq 300$ . Our models will only be useful for these quantities.]

Tickets Sold	50	100	200	300
Demand Price	\$120	\$115	\$80	\$65
Supply Price	\$45	\$65	\$70	\$80

For example, if Hank charges \$120 per ticket, he can expect to sell 50 tickets. On the other hand, he can obtain 50 of these tickets for \$2,250 (i.e. \$45 a piece).

- (a) Use the table of data to find supply and demand price functions. Use a **cubic** model for the demand function and a **quadratic** model for the supply function. [Use “Format trendline label” to show 6 decimal places of accuracy for the **cubic** model.]

Demand function:  $p_d =$  \_\_\_\_\_

Supply function:  $p_s =$  \_\_\_\_\_

The market equilibrium is  $(q_E, p_E) = ($  \_\_\_\_\_ , \_\_\_\_\_  $)$ .

[Round the quantity to 3 decimal places and the price to dollars and change. Stay inside  $0 \leq q \leq 300$ .]

If Hank charges \$90 per ticket, how many can he expect to sell? \_\_\_\_\_  
[Round to 3 decimal places.]

If Hank wants to sell 125 tickets, what will his (average supply) cost per ticket be? \$ \_\_\_\_\_

- (b) Use your model for the demand function to find a revenue function. Use your supply function to model the variable cost per bike and the fact that Hank has fixed costs of \$2,000 to find his cost function. Finally, use your revenue and cost functions to find profit, marginal revenue, marginal cost, and marginal profit functions.

Hank has \_\_\_\_\_ break even points. These occur at  $q =$  \_\_\_\_\_ tickets.  
[Round all quantities to 3 decimal places and remember to ignore answers outside  $0 \leq q \leq 300$ .]

Hank's 220<sup>th</sup> ticket brings in \$ \_\_\_\_\_ revenue. This ticket cost him \$ \_\_\_\_\_.

Hank's marginal profit is \$0 when  $q =$  \_\_\_\_\_ tickets.  
[List all quantities where  $MP(q) = 0$  and  $0 \leq q \leq 300$ . Round quantities to 3 decimal places.]

Hank will maximize his profit if he sells \_\_\_\_\_ tickets at a price of \$ \_\_\_\_\_ each.

His maximum possible profit is \$ \_\_\_\_\_.

- (c) Does Hank's profit function have a minimum? Either explain why it does not have one or if it does find where it is.

2. Use Excel to compute the following limits. If the limit does not exist write “DNE”.

(a) Let  $f(x) = (x - 3) \ln((x - 3)^2) + \frac{1}{x + 1}$

$$\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -1} f(x) = \underline{\hspace{2cm}}$$

(b)  $\lim_{x \rightarrow 0} e^{-1/x^2} = \underline{\hspace{2cm}}$

3. Recall that the **derivative** of  $f(x)$  is defined to be

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

where  $\frac{f(x + h) - f(x)}{h}$  is called the **difference quotient** of  $f(x)$ .

$$\text{Let } f(x) = \begin{cases} x \ln(x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

(a) Compute the difference quotient of  $f(x)$  when  $x = 3$  and  $h = 0.1$ .

$$\frac{f(3 + 0.1) - f(3)}{0.1} = \underline{\hspace{2cm}}$$

Now use Excel to compute the limit as  $h \rightarrow 0$ . This shows that  $f'(3) \approx \underline{\hspace{2cm}}$ .

(b) Use Excel to repeat the previous part for  $x = 0$  (try to compute  $f'(0)$ ). Does the limit ( $h \rightarrow 0$ ) of the difference quotient at  $x = 0$  exist? If it does exist, what is it? If it does not exist, why not? Explain your answer in a sentence or two.

[Be careful that  $f(0)$  is defined to be 0. The formula  $x \ln(x^2)$  is undefined there:  $0 \cdot \ln(0^2)$  is undefined because  $\ln(0)$  is undefined.]