Math 1030 Workshop #5 (Due: July 27th, 2016) **Name:** Answer Key

Please turn in a paper copy and **SHOW YOUR WORK!**

1. Consider the function $f(x) = \begin{cases} 3x^3+9x^2+5x+3 & x \le -1 \\ (7x^4-6x)e^{-x^2} & -1 < x \le 1 \\ x^2-5x+2\ln(x) & x > 1 \end{cases}$

Be careful! Wolfram Alpha may have trouble with this function. You might want to deal with it one piece at a time.

`piecewise[{ {3x^3+9x^2+5x+3,x<=-1}, {(7x^4-6x)e^(-x^2),-1<x<=1}, {x^2-5x+2 ln(x),x>1} }]` ✓

To find critical points we need to look at $f(x)$ one piece at a time. Before doing this, let's note that $f(x)$ has a critical points at $x=-1$ and $x=1$. $f(x)$ switches formulas and is discontinuous there. This means that $f'(x)$ is undefined at $x = \pm 1$ so these are critical points.

`derivative 3x^3+9x^2+5x+3` ✓

Alpha let's us know that this formula has roots at $x = -5/3$ and $-1/3$. Notice that $-5/3 \approx -1.667 \le -1$ but $-1/3 \approx -0.333 > -1$. This means that $x=-1/3$ is out of bounds for this formula (this formula is only valid for $x \le -1$), so we throw $x = -1/3$ out.

`derivative (7x^4-6x)e^(-x^2)` ✓

The middle formula's derivative has roots at $x \approx -1.23835$, 0.49781, and 1.56091. This formula is only valid for $-1 < x \le 1$, so we need to throw out -1.23835 and 1.56091.

`derivative x^2-5x+2 ln(x)` ✓

The derivative of our final piece has roots at $x=1/2$ and $x=2$. Again, this formula is only valid for $x>1$ so we must throw out $x=1/2$.

(a) Find all of the critical points of $f(x)$. $x =$ $-5/3 \approx -2.667$, -1, 0.49781, 1, and 2

(b) Restricting our attention to the interval $[-2,4]$...

`plot piecewise[{ {3x^3+9x^2+5x+3,x<=-1}, {(7x^4-6x)e^(-x^2),-1<x<=1}, {x^2-5x+2 ln(x),x>1} }] on [-2,4]` ✓

Looking at the plot we can see that the maximum value must occur at the first formula's peak (this occurs at its critical point at $x=-5/3 \approx -1.667$). The minimum value must occur at the third formula's valley (this occurs at its critical point at $x=2$). We just need to know the function's value at these points.

`piecewise[{ {3x^3+9x^2+5x+3,x<=-1}, {(7x^4-6x)e^(-x^2),-1<x<=1}, {x^2-5x+2 ln(x),x>1} }] at x=-5/3` ✓

Or... `3x^3+9x^2+5x+3 at x=-5/3` ✓

Either way, we get that $f(-5/3) = \dfrac{52}{9} \approx 5.77778$.

`piecewise[{ {3x^3+9x^2+5x+3,x<=-1}, {(7x^4-6x)e^(-x^2),-1<x<=1}, {x^2-5x+2 ln(x),x>1} }] at x=2` ✓

Or... `x^2-5x+2 ln(x) at x=2` ✓

Either way, we get that $f(2) = 2\ln(2) - 6 \approx -4.61371$.

The maximum value of $f(x)$ is $52/9 \approx 5.77778$. This occurs when $x =$ $-5/3 \approx -1.66667$.

The minimum value of $f(x)$ is $2\ln(2) - 6 \approx -4.61371$. This occurs when $x =$ 2.

2. Riverdale Highschool has an expensive copying machine. It could cost \$7,000 to replace their machine. They have recorded that they spent \$50 reparing the copier during its first year of operation. The next year it cost \$190 to fix their machine. The third year it cost \$350 to fix it.

   Use Excel to find a power model for the **average** annual repair costs. Then model the average annual cost of operating this copying machine using a function of the form: $A(t) = \frac{C}{t} + Rt^r$ where C is the cost of purchasing the copier and Rt^r models the repair costs.

   First, we need to figure out our power model for repair costs. When $t = 1$ we have that Rt^r should be \$50. For $t = 2$ our average repair costs are $\frac{\$50+\$190}{2} = \$120$ (per year). For $t = 3$ we get $\frac{\$50+\$190+\$350}{3} \approx \$196.67$. After creating a table in Excel with...

$t =$	1	2	3
$Rt^r =$	50	120	196.67

   ...insert a scatter plot. Then add a power trendline. Excel tells us that $y = 50.137x^{1.2483}$. Now we can write down our average annual cost function.

   $$A(t) = \underline{\frac{7000}{t} + 50.137t^{1.2483}}$$

   To find the minimum of $A(t)$ let's find its critical points. Then create a graph and see what the function looks like.

   ```
   derivative 7000/t + 50.137 t^1.2483 ✓
   ```

   ```
   plot 7000/t + 50.137 t^1.2483 on [0,20] ✓
   ```

   Clearly we have a critical point at $t = 0$ (where $A(t)$ is not continuous). But the only interesting critical point is at $t \approx 8.15054$. Looking at the plot, this is definitely the location of $A(t)$'s minimum. Note that $0.15054(12) = 1.80648 \approx 2$ months.

   ```
   7000/t + 50.137 t^1.2483 at t=8.15054 ✓
   ```

   When $t =$ __8.15054__, $A(t)$ is minimized. [Keep 5 decimal places.]

   Riverdale should replace its copying machine after __8__ years and __2__ months of opertion.
   [Round up to the next whole month.]

   If they do this, their average annual cost (for the copier) should be \$ __1,546.84__.

3. Stew's business uses specialty print cartridges for their photo ID printer each year. These cartridges cast \$150 a piece. Placing an order costs \$10. Stew's inventory cost is rather low. He estimates that it costs him \$0.10 per year per cartridge (based on average inventory with all of the standard assumptions). Let $C(x)$ be Stew's annual cost function for these cartridges.

   (a) If Stew needs 200 cartridges each year, $C(x) = \underline{200(150) + 10\left(\frac{200}{x}\right) + 0.10\left(\frac{x}{2}\right)}$

   ```
   plot 200(150) + 10(200/x) + 0.10(x/2) on [1,150] ✓
   ```

   ```
   derivative 200(150) + 10(200/x) + 0.10(x/2) ✓
   ```

   We see that $C(x)$ has critical points at ± 200 (and 0). Obviously, $x = 200$ is the extreme end of our domain. Our function decreases all the way down to the end. That is where our minimum occurs. [By the way, here our ideal solution is also a practical solution.]

   ```
   200(150) + 10(200/x) + 0.10(x/2) at x=200 ✓
   ```

   We see that $C(200) = 30,020$ is the minimum value of $C(x)$.

   His **ideal** EOQ is __200__ and his **ideal** minimum annual cost is __\$30,020__.

(b) If Stew needs 1,000 cartridges each year, $C(x) = $ $1000(150) + 10\left(\frac{1000}{x}\right) + 0.10\left(\frac{x}{2}\right)$

`plot 1000(150) + 10(1000/x) + 0.10(x/2) on [1,1000]` ✓

`derivative 1000(150) + 10(1000/x) + 0.10(x/2)` ✓

This time $C(x)$ has critical points at ± 447.214 (and 0). Looking at the graph, obviously the minimum occurs at $x = 447.214$. We need to know the value of $C(x)$ at this point.

`1000(150) + 10(1000/x) + 0.10(x/2) at x=447.214` ✓

We find that $C(447.214) \approx 150,045$. This is the minimum value of the cost function.

His **ideal** EOQ is 447.214 and his **ideal** minimum annual cost is $150,045 .

(c) Suppose that Stew needs 2,000 cartridges each year. In addition, he found out that he gets a discount if he places a large order. For orders of 300 or more, he pay $135 per cartridge. However, there is an additional delivery fee for these large orders. Instead of $10, it costs $50 to place a large order. His inventory costs stay the same.

$$C(x) = \begin{cases} 2000(150) + 10\left(\frac{2000}{x}\right) + 0.10\left(\frac{x}{2}\right) & x < 300 \\ 2000(135) + 50\left(\frac{2000}{x}\right) + 0.10\left(\frac{x}{2}\right) & x \geq 300 \end{cases}$$

`piecewise[{ {2000(150)+10(2000/x)+0.10(x/2),x<300}, {2000(135)+50(2000/x)+0.10(x/2),x>=300} }]` ✓

Clearly, the minimum occurs after the price break. Let's focus on that part of the formula.

`plot 2000(135)+50(2000/x)+0.10(x/2) on [300,2000]` ✓

It looks like the minimum occurs at this second formula's critical point (sound around 1400). Let's find its exact location.

`derivative 2000(135)+50(2000/x)+0.10(x/2)` ✓

Apparently this critical point is $x \approx 1414.21$. We need to plug this in to find the corresponding ideal minimum cost.

`2000(135)+50(2000/x)+0.10(x/2) at x=1414.21` ✓

Here we find the value of $C(x)$ at $x = 1414.21$ is 270141.

His **ideal** EOQ is 1,414.21 . His **ideal** minimum annual cost is $270,141 .