1. $f(x) = 3x^2 - \ln(x) + e^x + 5x - 2$
2. $f(x) = \sqrt{x} + \frac{2}{x} - 3x^5$
3. $y = x^3 e^x$
4. $y = \frac{x^2+1}{x^3-2x+1}$
5. $y = (\ln(x) + 2x - 7)^4$
6. $f(x) = \ln\left(\frac{x^2-2}{e^{3x}}\right)$
7. $y = \ln(4x^3(x^2+1))$
8. $f(x) = \frac{\sqrt{x}}{x^3} + e^{-2x+\ln(3x)}$
9. $f(x) = \ln(\sqrt{x^3 - x} + e^{5x})$
10. $y = e^{e^{e^x}}$
11. $y = x^3 \ln(x+3) e^{-x}$
12. $f(x) = \frac{\ln(3x+2)}{e^x + x}$
13. $y = \ln(\sqrt{x} e^{x^2 - 2x})$
14. $f(x) = (\ln(e^x + 1))^4$

Find an equation for the line tangent to $y = f(x)$ at the given point.

$y = x^2$ at $x = 2$

$y = \ln(x)$ at $x = 1$

$y = e^x$ at $x = 0$

$y = \sqrt{x}$ at $x = 9$

Answers:

1. $f(x) = 3x^2 - \ln(x) + e^x + 5x - 2 \implies f'(x) = 6x - \frac{1}{x} + e^x + 5$

2. $f(x) = \sqrt{x} + \frac{2}{x} - 3x^5 \implies f(x) = x^{1/2} + 2x^{-1} - 3x^5 \implies$
$f'(x) = \frac{1}{2}x^{-1/2} - 2x^{-2} - 15x^4$

3. $y = x^3 e^x \implies y' = 3x^2 e^x + x^3 e^x$

4. $y = \frac{x^2+1}{x^3-2x+1} \implies y' = \frac{2x(x^3-2x+1) - (x^2+1)(3x^2-2)}{(x^3-2x+1)^2} \implies$
$y' = \frac{2x^4 - 4x^2 + 2x - 3x^4 - 3x^2 + 2x^2 + 2}{(x^3-2x+1)^2} \implies y' = \frac{-x^4 - 5x^2 + 2x + 2}{(x^3-2x+1)^2}$

5. $y = (\ln(x) + 2x - 7)^4 \implies y' = 4(\ln(x) + 2x - 7)^3 \left(\frac{1}{x} + 2\right)$

6. $f(x) = \ln\left(\frac{x^2-2}{e^{3x}}\right) \implies f(x) = \ln(x^2-2) - \ln(e^{3x}) \implies$
$f(x) = \ln(x^2-2) - 3x \implies f'(x) = \frac{2x}{x^2-2} - 3$

7. $y = \ln(4x^3(x^2+1)) \implies y = \ln(4) + \ln(x^3) + \ln(x^2+1) \implies$
$y = \ln(4) + 3\ln(x) + \ln(x^2+1) \implies y' = \frac{3}{x} + \frac{2x}{x^2+1}$

8. $f(x) = \frac{\sqrt{x}}{x^3} + e^{-2x+\ln(3x)} \implies f(x) = x^{1/2}x^{-3} + e^{-2x}e^{\ln(3x)} \implies$
$f(x) = x^{-5/2} + e^{-2x}3x \implies f'(x) = -\frac{5}{2}x^{-7/2} + e^{-2x}(-2)3x + e^{-2x}3 \implies$
$f'(x) = -\frac{5}{2}x^{-7/2} - 6xe^{-2x} + 3e^{-2x}$

9. $f(x) = \ln(\sqrt{x^3 - x} + e^{5x}) \implies f(x) = \ln((x^3-x)^{1/2} + e^{5x}) \implies$
$f'(x) = \frac{1}{(x^3-x)^{1/2} + e^{5x}} \left(\frac{1}{2}(x^3-x)^{-1/2}(3x^2-1) + e^{5x}5\right)$

10. $y = e^{e^{e^x}} \implies y' = e^{e^{e^x}} e^{e^x} e^x$

11. $y = x^3 \ln(x+3)e^{-x} \implies y' = 3x^2 \ln(x+3)e^{-x} + x^3 \frac{1}{x+3} e^{-x} + x^3 \ln(x+3)e^{-x}(-1)$

12. $f(x) = \frac{\ln(3x+2)}{e^x + x} \implies f'(x) = \frac{\frac{3}{3x+2}(e^x + x) - \ln(3x+2)(e^x+1)}{(e^x+x)^2}$

13. $y = \ln(\sqrt{x} e^{x^2-2x}) \implies y = \ln(x^{1/2}) + \ln(e^{x^2-2x}) \implies y = \frac{1}{2}\ln(x) + x^2 - 2x \implies y' = \frac{1}{2x} + 2x - 2$

14. $f(x) = (\ln(e^x+1))^4 \implies f'(x) = 4(\ln(e^x+1))^3 \frac{1}{e^x+1} e^x$

Find an equation for the line tangent to $y = f(x)$ at the given point.

- $y = x^2$ at $x = 2$.
  Notice that $y' = 2x$. The slope of the tangent of $y = x^2$ at $x = 2$ is
  $y'|_{x=2} = 2x|_{x=2} = 2(2) = 4$. If $x = 2$, then $y = 2^2 = 4$.
  Using point slope: $y - 4 = 4(x - 2)$.
  The equation of the tangent is $y = 4x - 4$.

- $y = \ln(x)$ at $x = 1$.
  Notice that $y' = 1/x$. The slope of the tangent of $y = \ln(x)$ at $x = 1$ is
  $y'|_{x=1} = \left.\frac{1}{x}\right|_{x=1} = \frac{1}{1} = 1$. If $x = 1$, then $y = \ln(1) = 0$.
  Using point slope: $y - 0 = 1(x - 1)$.
  The equation of the tangent is $y = x - 1$.

- $y = e^x$ at $x = 0$.
  Notice that $y' = e^x$. The slope of the tangent of $y = e^x$ at $x = 0$ is
  $y'|_{x=0} = e^x|_{x=0} = e^0 = 1$.
  If $x = 0$, then $y = e^0 = 1$. Using point slope: $y - 1 = 1(x - 0)$. The equation of the tangent is $y = x + 1$.

- $y = \sqrt{x} = x^{1/2}$ at $x = 9$.
  Notice that $y' = (1/2)x^{-1/2}$. The slope of the tangent of $y = x^{1/2}$ at $x = 9$ is $y'|_{x=9} = \left.(1/2)x^{-1/2}\right|_{x=9} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$.
  If $x = 9$, then $y = \sqrt{9} = 3$.
  Using point slope: $y - 3 = (1/6)(x - 3)$.
  The equation of the tangent is $y = \frac{1}{6}x + \frac{3}{2}$.