1. $\displaystyle\int 3x^2 - \frac{1}{x} + e^x - 7\,dx$

2. $\displaystyle\int \frac{\sqrt{x}}{x^3} + \frac{6+2x}{x^2} - 50x^{99}\,dx$

3. $\displaystyle\int 3\sin(x) + 10\sec(x)\tan(x)\,dx$

4. $\displaystyle\int \frac{5}{\sqrt{1-x^2}}\,dx$

5. $\displaystyle\int dx$

6. $\displaystyle\int \frac{e^{5x}}{e^{2x}} - 8x^{7/3}\,dx$

7. $\displaystyle\int (x^2-5x+6)^9(2x-5)\,dx$

8. $\displaystyle\int e^{6x-2}\,dx$

9. $\displaystyle\int \cos(5x+1)\,dx$

10. $\displaystyle\int x\sec^2(x^2)\,dx$

11. $\displaystyle\int \frac{x}{x^2+1}\,dx$

12. $\displaystyle\int \frac{1}{x^2+1}\,dx$

13. $\displaystyle\int \frac{x+3}{x^2+6x-7}\,dx$

14. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

15. $\displaystyle\int \frac{\ln(x)}{x}\,dx$

16. $\displaystyle\int e^{e^x}e^x\,dx$

17. Solve the initial value problem $y' = \cos(x) + e^x + 1$ where $y(0) = 3$.

18. Find the antiderivative $f(x)$ of $g(x) = \dfrac{500}{\sqrt{2x+1}} + 6x^2 + 3$ such that $f(4) = 10$.

Answers:

1. $\displaystyle\int 3x^2 - \frac{1}{x} + e^x - 7\,dx = x^3 - \ln|x| + e^x - 7x + C$

2. $\displaystyle\int \frac{\sqrt{x}}{x^3} + \frac{6+2x}{x^2} - 50x^{99}\,dx = \int x^{-3+1/2} + \frac{6}{x^2} + \frac{2x}{x^2} - 50x^{99}\,dx$
$\displaystyle= \int x^{-5/2} + 6x^{-2} + \frac{2}{x} - 50x^{99}\,dx = \frac{x^{-3/2}}{-3/2} + 6\frac{x^{-1}}{-1} + 2\ln|x| - 50\frac{x^{100}}{100} + C = -\frac{2}{3x^{3/2}} - \frac{6}{x} + 2\ln|x| - \frac{1}{2}x^{100} + C$

3. $\displaystyle\int 3\sin(x) + 10\sec(x)\tan(x)\,dx = -3\cos(x) + 10\sec(x) + C$

4. $\displaystyle\frac{5}{\sqrt{1-x^2}}\,dx = 5\arcsin(x) + C$

5. $\displaystyle\int dx = x + C$

6. $\displaystyle\int \frac{e^{5x}}{e^{2x}} - 8x^{7/3}\,dx = \int e^{5x-2x} - 8x^{7/3}\,dx = \int e^{3x} - 8x^{7/3}\,dx = \frac{1}{3}e^{3x} - 8\frac{x^{10/3}}{10/3} + C = \frac{e^{3x}}{3} - \frac{24}{10}x^{10/3} + C$

7. $\displaystyle\int (x^2-5x+6)^9(2x-5)\,dx$ Substitute $u = x^2-5x+6$ so that $du = (2x-5)\,dx$.
So our integral becomes $\displaystyle\int u^9\,du = \frac{u^{10}}{10} + C = \frac{(x^2-5x+6)^{10}}{10}$

8. $\int e^{6x-2}\,dx$ Substitute $u = 6x - 2$ so that $du = 6\,dx$ thus $(1/6)\,du = dx$.

So our integral becomes $\int e^u \frac{1}{6}\,du = \frac{1}{6}e^u + C = \frac{e^{6x-2}}{6} + C$

9. $\int \cos(5x+1)\,dx$ Substitute $u = 5x + 1$ so that $du = 5\,dx$ thus $(1/5)\,du = dx$.

So our integral becomes $\int \cos(u)\frac{1}{5}\,du = \frac{1}{5}\sin(u) + C = \frac{1}{5}\sin(5x+1) + C$

10. $\int x\sec^2(x^2)\,dx$ Substitute $u = x^2$ so that $du = 2x\,dx$ thus $(1/2)\,du = dx$.

So our integral becomes $\int \sec^2(u)\frac{1}{2}\,du = \frac{1}{2}\tan(u) + C = \frac{1}{2}\tan(x^2) + C$

11. $\int \frac{x}{x^2+1}\,dx$ Substitute $u = x^2 + 1$ so that $du = 2x\,dx$ thus $(1/2)\,du = dx$

So our integral becomes $\int \frac{1}{u} \cdot \frac{1}{2}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+1) + C$

12. $\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$ [Just a basic formula here!]

13. $\int \frac{x+3}{x^2+6x-7}\,dx$ Substitute $u = x^2 + 6x - 7$ so that $du = (2x+6)\,dx = 2(x+3)\,dx$ thus $(1/2)\,du = (x+3)dx$.

So our integral becomes $\int \frac{(1/2)\,du}{u} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+6x-7| + C$

14. $\int \frac{e^{1/x}}{x^2}\,dx = \int e^{x^{-1}}x^{-2}\,dx$ Substitute $u = x^{-1}$ so that $du = -x^{-2}\,dx$ thus $-du = x^{-2}\,dx$.

So our integral becomes $\int e^u(-du) = -e^u + C = -e^{1/x} + C$

15. $\int \frac{\ln(x)}{x}\,dx = \int \ln(x)\frac{1}{x}\,dx$ Substitute $u = \ln|x|$ so that $du = (1/x)\,dx$.

So our integral becomes $\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln|x|)^2 + C$

16. $\int e^{e^x}e^x\,dx$ Substitute $u = e^x$ so that $du = e^x\,dx$. So our integral becomes $\int e^u\,du = e^u + C = e^{e^x} + C$

17. Solve the initial value problem $y' = \cos(x) + e^x + 1$ where $y(0) = 3$.

$y = \int y'\,dx = \int \cos(x) + e^x + 1\,dx = \sin(x) + e^x + x + C$.

We require $3 = y(0) = \sin(0) + e^0 + 0 + C = 1 + C$ so $C = 2$. Thus $y = \sin(x) + e^x + x + 2$.

18. Find the antiderivative $f(x)$ of $g(x) = \frac{500}{\sqrt{2x+1}} + 6x^2 + 3$ such that $f(4) = 10$.

$f(x) = \int g(x)\,dx = \int 500(2x+1)^{-1/2} + 6x^2 + 3\,dx = 500(2x+1)^{1/2} + 2x^3 + 3x + C$ (For the term "$500(2x+1)^{-1/2}$" use the substitution $u = 2x + 1$). We also know that $f(4) = 10$ so $10 = f(4) = 500(2(4)+1)^{1/2} + 2(4^3) + 3(4) + C$ $10 = 500\sqrt{9} + 128 + 12 + C$ so $10 = 1500 + 140 + C$ so $C = -1630$.

Therefore, $f(x) = 500\sqrt{2x+1} + 2x^3 + 3x - 1630$