Math 1110

Some Trigonometric Formulas

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- Pythagorean Theorem: $\sin^2(x) + \cos^2(x) = 1$
- Double angle: $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$, $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\sin(2x) = 2\sin(x)\cos(x)$
- Angle sum and difference: $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$, $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$,
  $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$, $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- Product to sum: $2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$, $2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$,
  $2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$

Occasionally we need trigonometric identities to derive derivative formulas, simplify expressions, or work through certain word problems. The Pythagorean Theorem is by far the most important and most frequently used identity. The remaining identities become more important in Calculus 2 where the focus is on undoing differentiation (i.e., integration).

Here is a run down of some helpful facts and definitions:

- Cosine is even: $\cos(-x) = \cos(x)$ Sine is odd: $\sin(-x) = -\sin(x)$
- Pythagorean theorem (and consequences): $\sin^2(x) + \cos^2(x) = 1$, $\tan^2(x) + 1 = \sec^2(x)$, $\tan^2(x) = \sec^2(x) - 1$
  Less used: $1 + \cot^2(x) = \csc^2(x)$, $\cot^2(x) = \csc^2(x) - 1$
- Definitions: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, $\sec(x) = \dfrac{1}{\cos(x)}$, $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$, $\csc(x) = \dfrac{1}{\sin(x)}$
  $$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \text{tagent} = \frac{\text{opposite}}{\text{adjacent}},$$
  $$\text{cosecant} = \frac{\text{hypotenuse}}{\text{opposite}}, \quad \text{secant} = \frac{\text{hypotenuse}}{\text{adjacent}}, \quad \text{cotagent} = \frac{\text{adjacent}}{\text{opposite}}$$
- Special values: $\sin(0) = 0$, $\sin(\pi/2) = 1$, $\sin(\pi) = 0$, $\sin(3\pi/2) = -1$, $\sin(2\pi) = 0$, etc. (draw sine's graph)
  $\cos(0) = 1$, $\cos(\pi/2) = 0$, $\cos(\pi) = -1$, $\cos(3\pi/2) = 0$, $\cos(2\pi) = 1$, etc. (draw cosine's graph)
  $\sin(\pi/4) = \cos(\pi/4) = \dfrac{1}{\sqrt{2}}$ and $\tan(\pi/4) = 1$ (draw $45° - 45° - 90°$ special triangle)
  $\sin(\pi/3) = \cos(\pi/6) = \dfrac{\sqrt{3}}{2}$, $\cos(\pi/3) = \sin(\pi/6) = \dfrac{1}{2}$, etc. (draw $30° - 60° - 90°$ special triangle)
- We frequently use derivatives of sine and cosine, as well as $\dfrac{d}{dx}\Big[\tan(x)\Big] = \sec^2(x)$ and $\dfrac{d}{dx}\Big[\sec(x)\Big] = \sec(x)\tan(x)$.

  We less frequently use $\dfrac{d}{dx}\Big[\cot(x)\Big] = -\csc^2(x)$ and $\dfrac{d}{dx}\Big[\csc(x)\Big] = -\csc(x)\cot(x)$.
- Trigonometric functions are not strictly invertible. However, by restricting domains we can choose portions to invert yielding inverse "branches". For $\sin(\theta)$ one restricts inputs to the interval $[-\pi/2, \pi/2]$ so that $\arcsin(x)$ has outputs in that interval $[-\pi/2, \pi/2]$. The same is choice is made for $\arctan(x)$. On the other hand, $\arccos(x)$ and $\text{arccot}(x)$ output angles in the interval $[0, \pi]$.

  For example, $\cdots = \cos(-3\pi) = \cos(-\pi) = \cos(\pi) = \cos(3\pi) = \cdots = -1$, so infinitely many angles yield a cosine of -1. However, $\arccos(-1) = \pi$ since π is the only one of those angles belonging to the interval $[0, \pi]$. Likewise, $\cdots = \tan(5\pi/4) = \tan(\pi/4) = \tan(-3\pi/4) = \cdots = 1$, but $\arctan(1) = \pi/4$ since $\pi/4$ is the only angle belonging to $[-\pi/2, \pi/2]$

  Inverse secant and cosecant functions are a bit trickier. Our textbook picks branches of these inverses so that $\text{arcsec}(x)$ outputs angles in the intervals $[0, \pi/2) \cup [\pi, 3\pi/2)$ and $\text{arccsc}(x)$ outputs angles in the intervals $(-\pi, -\pi/2] \cup (0, \pi/2]$, but this is somewhat non-standard. Many texts choose $\text{arcsec}(x)$ to output in $[0, \pi/2) \cup (\pi/2, \pi]$ and $\text{arccsc}(x)$ to output in $[-\pi/2, 0) \cup (0, \pi/2]$. We will not worry about these conventions and just avoid these functions.