Math 1110-101 **Homework #4** **Due: Thurs., August 3^{rd}, 2023**

#1 Differential Equations

(a) Verify that $y = te^{-3t}$ is a solution of $y'' + 6y' + 9y = 0$.

(b) Solve the initial value problem: $y' = \dfrac{x^2+1}{x^3} + 2$ and $y(1) = 5$.

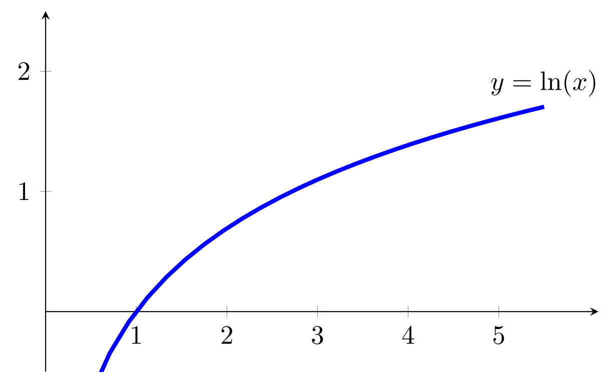
Hint: Do some algebra to simplify your fraction first.

#2 Approximate!

Recall that L_n, R_n, and M_n are the left, right, and midpoint rule approximations of an integral when using a regular (=equally spaced) partition of n subintervals. In addition, $T_n = \frac{1}{2}L_n + \frac{1}{2}R_n$ approximates our integral using n trapezoids. Finally, $S_{2n} = \frac{2}{3}M_n + \frac{1}{3}T_n$ approximates using parabolic arcs.

(a) Consider the definite integral $\displaystyle\int_{-3}^{9} \sqrt[3]{x^2}\,dx$. Compute L_3, R_3, M_3, T_3, and S_6 (=$S_{2\cdot 3}$).

(b) Suppose $I = \displaystyle\int_1^5 \ln(x)\,dx$. Sketch a picture of the approximations R_2 and T_2 for this integral.



(c) Still considering $I = \displaystyle\int_1^5 \ln(x)\,dx$, rank L_n, R_n, M_n, T_n, I.

[For example, $L_n \leq R_n \leq M_n \leq T_n \leq I$ is definitely not the correct answer!]

#3 Exact area.

(a) First, carefully sketch $y = 2x + 2$ on the interval $[-2, 3]$.

Then, compute $\displaystyle\int_{-2}^{3} (2x+2)\,dx$ using basic geometry (i.e., interpret the integral as net area).

(b) First, carefully sketch $y = \sqrt{16 - x^2}$ (*Hint:* this is the upper-half of a circle).

Then, compute $\displaystyle\int_{-4}^{4} \sqrt{16 - x^2}\,dx$ using basic geometry.