Math 1120 # Test #2 – Formulas **Sept. 29^{th}, 2021**

Sections 7.2–7.4 rely heavily on trigonometric identities. I will provide the following identities on Test #2:

- Double angle: $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$, $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$, $\sin(2x) = 2\sin(x)\cos(x)$
- Angle sum and difference: $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$, $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$, $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$, $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- Product to sum: $2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$, $2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$, $2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$

The first two double angle identities are used to handle integrals of purely even powers of sine and cosine. These first show up in section 7.2. The third double angle identity shows up at the end of section 7.3 problems when converting back from something like $\sin(2\theta)$. We don't tend to need the angle sum and difference formulas directly. The product-to-sum identities showed up in problems at the end of section 7.2.

Stuff you should know (not provided on Test #2):

- Cosine is even: $\cos(-x) = \cos(x)$ Sine is odd: $\sin(-x) = -\sin(x)$
- Pythagorean theorem (and consequences): $\sin^2(x) + \cos^2(x) = 1$, $\tan^2(x) + 1 = \sec^2(x)$, $\tan^2(x) = \sec^2(x) - 1$

  Less used: $1 + \cot^2(x) = \csc^2(x)$, $\cot^2(x) = \csc^2(x) - 1$

  One use of these identities is to do (inverse) trigonometric substitutions (this is section 7.3):

  Since $\cos^2(\theta) = 1 - \sin^2(\theta)$, we can use $x = a \cdot \sin(\theta)$ to get $a^2 - x^2 = a^2 - a^2\sin^2(\theta) = a^2\cos^2(\theta)$.

  Since $\tan^2(\theta) = \sec^2(\theta) - 1$, we can use $x = a \cdot \sec(\theta)$ to get $x^2 - a^2 = a^2\sec^2(\theta) - a^2 = a^2\tan^2(\theta)$.

  Since $\sec^2(\theta) = \tan^2(\theta) + 1$, we can use $x = a \cdot \tan(\theta)$ to get $x^2 + a^2 = a^2\tan^2(x) + a^2 = a^2\sec^2(\theta)$.
- Definitions: $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$, $\cot(x) = \frac{\cos(x)}{\sin(x)}$, $\csc(x) = \frac{1}{\sin(x)}$

  $\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}$, $\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\text{tagent} = \frac{\text{opposite}}{\text{adjacent}}$,

  $\text{cosecant} = \frac{\text{hypotenuse}}{\text{opposite}}$, $\text{secant} = \frac{\text{hypotenuse}}{\text{adjacent}}$, $\text{cotagent} = \frac{\text{adjacent}}{\text{opposite}}$
- Special values: $\sin(0) = 0$, $\sin(\pi/2) = 1$, $\sin(\pi) = 0$, $\sin(3\pi/2) = -1$, $\sin(2\pi) = 0$, etc. (draw sine's graph)

  $\cos(0) = 1$, $\cos(\pi/2) = 0$, $\cos(\pi) = -1$, $\cos(3\pi/2) = 0$, $\cos(2\pi) = 1$, etc. (draw cosine's graph)

  $\sin(\pi/4) = \cos(\pi/4) = \frac{1}{\sqrt{2}}$ and $\tan(\pi/4) = 1$ (draw $45^\circ - 45^\circ - 90^\circ$ special triangle)

  $\sin(\pi/3) = \cos(\pi/6) = \frac{\sqrt{3}}{2}$, $\cos(\pi/3) = \sin(\pi/6) = \frac{1}{2}$, etc. (draw $30^\circ - 60^\circ - 90^\circ$ special triangle)
- We frequently use derivatives of sine and cosine, as well as $\frac{d}{dx}\Big[\tan(x)\Big] = \sec^2(x)$ and $\frac{d}{dx}\Big[\sec(x)\Big] = \sec(x)\tan(x)$.

  We also frequently use $\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$ and $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$

  We less frequently use $\frac{d}{dx}\Big[\cot(x)\Big] = -\csc^2(x)$ and $\frac{d}{dx}\Big[\csc(x)\Big] = -\csc(x)\cot(x)$.

  The textbook sometimes uses: $\int \sec(x) = \ln|\sec(x) + \tan(x)| + C$ and $\int \csc(x) = -\ln|\csc(x) + \cot(x)| + C$. You do not need to know these last integrals.