For each of the following series apply a convergence/divergence test to determine whether the series converges or diverges.

Blanket Statement: In each case, these series have (eventually) positive terms, so all of our tests are potentially valid to apply (comparison, integral, and ratio).

1. $\displaystyle\sum_{k=1}^{\infty} \ln(k)$

   This series **diverges**. The easiest way to see this is: $\lim_{k\to\infty} \ln(k) = \infty \neq 0$. Since the limit of the terms being summed is not zero the series cannot converge.

   A second approach would be to use a comparison. Notice that $0 < 1 < \ln(k)$ (at least this is true when $k \geq 3$). Then since $\sum_{k=1}^{\infty} 1$ diverges (limit of the terms is $1 \neq 0$), the sum of logs diverges.

   Please note that the ratio test does work here. The limit of the ratio of terms is 1 (you'll need L'Hopitals rule to evaluate the limit), so the ratio test says nothing.

2. $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{k^2+5}$

   This series **diverges**. The numerator is an exponential function and the denominator is a polynomial, so the limit of the terms: $\displaystyle\lim_{k\to\infty} \frac{2^k}{k^2+5} = \infty \neq 0$ so the series diverges.

   Although the above argument works, it might be easier to use the ratio test (especially if that limit doesn't seem obvious to you).

$$\lim_{k\to\infty} \frac{\frac{2^{k+1}}{(k+1)^2+5}}{\frac{2^k}{k^2+5}} = \lim_{k\to\infty} \frac{2^{k+1}(k^2+5)}{((k+1)^2+5)2^k} = \lim_{k\to\infty} \frac{2(k^2+5)}{(k+1)^2+5} = 2 > 1$$

   So the series diverges by the ratio test.

3. $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{k!}$

   This series **converges**. Use the ratio test.

$$\lim_{k\to\infty} \frac{\frac{5^{k+1}}{(k+1)!}}{\frac{5^k}{k!}} = \lim_{k\to\infty} \frac{5^{k+1}k!}{(k+1)!5^k} = \lim_{k\to\infty} \frac{\cancel{5^k}\,5\cancel{k!}}{\cancel{5^k}\,(k+1)\cancel{k!}} = \lim_{k\to\infty} \frac{5}{k+1} = 0 < 1$$

   So the series converges by the ratio test.

4. $\displaystyle\sum_{k=0}^{\infty} \frac{k^2+3k+1}{2k^4+9k^3+5}$

This series **converges**. Ignoring all of the "lower order" terms, it looks like the p-series $\sum 1/k^2$ which converges since $p = 2 > 1$. Since we suspect the series converges and it is *comparable* to a p-series, we should use the comparison test.

$$0 \leq \frac{k^2+3k+1}{2k^4+9k^3+5} \leq \frac{k^2+3k^2+k^2}{2k^4} = \frac{5}{2} \cdot \frac{k^2}{k^4} = \frac{5}{2} \cdot \frac{1}{k^2}$$

(The inequality holds since we made the numerator *bigger* and the denominator *smaller*.) Next, we note that $\frac{5}{2}\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges (since it's a multiple of a convergent p-series). Therefore, our series converges (by the comparison test).

Note: To show that a series converges using the comparison test we must find an **upper bound** coming from a **convergent** series. To show divergence we find a **lower bound** coming from a **divergent** series.