

We will use mathematical induction to prove the following familiar proposition of Euclidean geometry:

Proposition For $n \geq 3$, the sum of the measure of the interior angles of a convex polygon with n vertices is $(n - 2)\pi$ radians.

Proof: We will proceed by induction.

Base Case ($n = 3$): A (convex) polygon with 3 vertices is just a triangle. We know that the sum of the measures of the angles in a triangle is $\pi = (3 - 2)\pi$ radians. Thus our proposition holds for $n = 3$.

Inductive Step: Fix some $n \geq 3$ and assume the the proposition holds for this n . Then consider a convex polygon with $n + 1$ vertices. We provide a figure below:

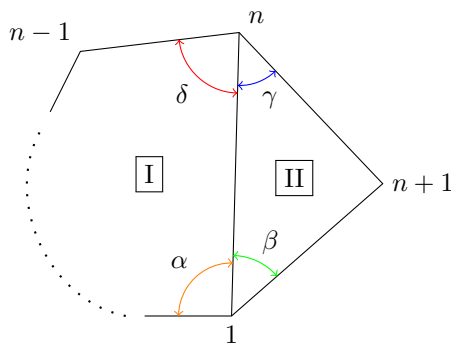


Figure: Our convex polygon with $n + 1$ vertices.

As done in the above figure, draw a line segment between vertex 1 and vertex n . Since the polygon is convex, this segment will lie *inside* the polygon. We now have two convex polygons made coming from our original convex polygon. Let us label the polygon with vertices 1 through n as **I** and the polygon with vertices n , $n + 1$, and 1 as **II**.

By our inductive hypothesis the sum of the measures of the angles of polygon **I** is $(n - 2)\pi$ radians. On the other hand, polygon **II** is a triangle, so its angles sum to π radians.

Notice that the measure of the angle at vertex n in our original $(n + 1)$ vertex polygon is the sum of the angles at vertex n in polygon **I** and polygon **II** (i.e., the angle at n is $\delta + \gamma$). Likewise, the measure of the angle at vertex 1 in our original polygon is the sum of the angles of polygons **I** and **II** at vertex 1 (i.e., the angle at 1 is $\alpha + \beta$).

The measures of the angles in our original polygon at vertices 2 through $n - 1$ are the same as those in polygon **I**. Likewise, the measure of the angle at vertex $n + 1$ is the same in our original polygon as it is the the triangle **II**.

Thus the sum of the measures of the angles at the vertices of our original polygon is equal to the sum of the angles of polygons **I** plus the sum of the angles of our triangle **II**. Therefore, the sum of the measures of the angles of our original polygon is $(n - 2)\pi + \pi = (n - 1)\pi = ((n + 1) - 2)\pi$ radians.

We have just shown that if the proposition holds for convex polygons with n vertices, then it must also hold for convex polygons with $n + 1$ vertices.

Therefore, by induction our proposition holds for all $n \geq 3$. ■