Math 2110 | # Homework #1 | **Due: Wed., Aug. 26^{th}, 2020**

1. Consider the following statement: "Bill is old, but his wife is not." Give appropriate definitions for a and b and translate our statement into a formal proposition in terms of a and b. Define and translate in such a way that your logical statement involves at least one negation.

2. Let a be the statement "Tomorrow is Thursday", b be "The sky is blue", and c be "Becky loves chocolate". Write down a plain English version of the proposition using these translations of a, b, and c.

   (a) Proposition #1: $a \vee \neg b$

   (b) Proposition #2: $(a \wedge \neg b) \to c$

   (c) Proposition #3: $\neg a \wedge (b \to c)$

3. For each of the following propositions,

   I. Write down an abbreviated truth table.

   II. State whether the proposition is a tautology, contradiction, or contingency.

   (a) Proposition #1: $a \to (\neg b \to a)$

   (b) Proposition #2: $(a \wedge b) \to (a \leftrightarrow c)$

   (c) Proposition #3: $\neg(a \vee b) \wedge (a \to \neg b)$

4. The Sheffer stroke[1] or NAND (= not and) operation is denoted $a \uparrow b$ and has the following truth table:

a	b	$a \uparrow b$
T	T	F
F	T	T
T	F	T
F	F	T

   Show that $a \uparrow b$ is logically equivalent to $\neg(a \wedge b)$.

   Note: It turns out that the Sheffer stroke is "sufficient" for expressing all possible propositions. This follows if our typical operations are already sufficient (they are – in fact just negation and disjunction are enough) and we know how to translate our usual operations in terms of Sheffer stokes – which can be done as follows:

$$\neg a \longleftrightarrow (a \uparrow a) \qquad (a \vee b) \longleftrightarrow ((a \uparrow a) \uparrow (b \uparrow b)) \qquad (a \wedge b) \longleftrightarrow ((a \uparrow b) \uparrow (a \uparrow b))$$

$$(a \to b) \longleftrightarrow (a \uparrow (b \uparrow b)) \qquad (a \leftrightarrow b) \longleftrightarrow ((a \uparrow b) \uparrow ((a \uparrow a) \uparrow (b \uparrow b)))$$

[1] See for example: https://en.wikipedia.org/wiki/Sheffer_stroke