Name: My Name Goes Here

Math 2110

Please use  $IAT_EX$  to type up this homework set!

#1 I have provided the proof of Lemma 7 - it's just missing justifications for each line. Please fill in the justifications. **Lemma 7:**  $A \to (B \to C) \vdash_L B \to (A \to C)$ 

1: $A \to (B \to C)$	<u>1: Given</u>
2: $B \to (A \to (B \to C))$	<u>2: ?????</u>
3: $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	<u>3: ?????</u>
4: $(A \to B) \to (A \to C)$	<u>4: ?????</u>
5: $B \to ((A \to B) \to (A \to C))$	<u>5: ?????</u>
6: $B \to (A \to B)$	<u>6: ?????</u>
7: $B \to (A \to C)$	<u>7: ?????</u>

#2 Prove the following lemmas (without using the deduction theorem): L8 and L12.

**Lemma 8:**  $A \to B, B \to C \vdash_L A \to C$ **Lemma 12:**  $\vdash_L B \to \neg \neg B$ 

#3 Prove the following lemmas (you may use the deduction theorem if you want): L15, L22, and L24.

Lemma 15:	$\vdash_L \neg A \to (A \to B)$
Lemma 22:	$B \vdash_L A \lor B$
Lemma 24:	$A \leftrightarrow B \vdash_L B \to A$

#4 Consider the statements:  $A \to B, B \vdash_L A$  and  $A, \neg B \vdash_L \neg (A \land B)$ .

One of these is actually a theorem of L. The other statement is not. Prove this is the case.

[Hint/suggestion: Truth tables should help. Don't try to find System L proofs.]

**NOTE:** While you should create truth tables for yourself, I won't require that you type them up. You can merely summarize what they tell you. You can say something like "The truth table of XXXX showed that this statement was a YYYY. Therefore, ZZZZ."

Template Proof Code: If we use the Deduction Theorem our proof looks something like this...

Consider  $stuff \vdash_L stuff$ . **Proof:** 

1:	???	<u>1:</u>	???
2:	???	2:	???

By the Deduction Theorem blah blah.

Otherwise, our proof could just look like this...

1:	???	<u>1:</u>	???
2:	???	2:	???