Math 2110 **Homework #3** **Due: Fri., Sept. 11^{th}, 2020**

NOTE: You are always welcome to type up your homework. However, you are not required to type up any of this particular assignment. You can turn in (neatly) handwritten work.

1. Consider the statement $A(f(\underline{c})) \to ((\exists x \, \forall y \, B(x, y)) \wedge (\exists z \, C(z)))$.

   We use a model where the universe is all people, $A(x)$ is defined by "x does not age", $B(x, y)$ is defined by "x is always excellent towards y", $C(x)$ is defined by "x parties on forever", $\underline{c}$ is "Ava Syme-Reeves", and $f(y)$ is defined to be "the father of y".

   Using the above model, translate our statement to English.

   Note: Don't try to evaluate whether the statement is true or not. It may or may not make sense.

2. Consider the statement $\forall x \, (A(x, f(x, \underline{c})) \wedge B(x, y)) \to \exists z \, (C(w, z) \vee D(z))$.

   (a) Underline the scope of each quantifier.

   (b) Indicate which variables are bound and which ones are free. Is this a sentence? (Why or why not?)

   (c) Consider the term $t = g(x, \underline{a})$. Is t free for y in the above statement? (Why or why not?)

   (d) Consider the term $t = g(y, \underline{a})$. Is t free for y in the above statement? (Why or why not?)

3. Consider the sentence $\forall x \, \exists y \, (A(x, y) \to (B(f(y)) \wedge C(\underline{a})))$.

   (a) Build a model (using $\mathbb{N} = \{0, 1, 2, \dots\}$ as the universe) where this sentence is true.

   (b) Build a model (using $\mathbb{N} = \{0, 1, 2, \dots\}$ as the universe) where this sentence is false.

   (c) Is this sentence logically valid? (Why or why not?)