Math 2110 | # Homework #6 | **Due: Wed., Oct. 7^{th}, 2020**

Handwritten solutions are fine. ☺

#1 Let n be a natural number and suppose n^2 is odd. Prove that n is odd.

#2 Prove that there is no largest natural number.

#3 Prove that $\log_{10}(2)$ is irrational. *Hint:* The Fundamental Theorem of Arithmetic (integers greater than one have *unique* prime factorizations) should help at some point in your proof.

#4 Prove Theorem 20 (page 80): Let p be a natural number bigger than 1. If there is no natural number m dividing p such that $1 < m \leq \sqrt{p}$, then p is prime.

#5 Suppose that $r \in \mathbb{R}$ (i.e., r is a real number). It is the case that there is some $n \in \mathbb{Z}$ (i.e., integer n) such that $n \leq r < n+1$ (you don't have to prove this).[1] Prove that such an integer is unique (i.e., there is only one integer n such that $n \leq r < n+1$).

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[1] **Existence Proof:** Let $n = \lfloor r \rfloor$ where $\lfloor r \rfloor$ is the floor function (i.e., the largest integer less than or equal to r). When $r \geq 0$, $\lfloor r \rfloor$ is just the integer part of r (i.e., $\lfloor n.d_1d_2d_3\cdots \rfloor = n$) and if $r < 0$, then $\lfloor r \rfloor$ is the integer part minus 1 (e.g., $\lfloor -3.134 \rfloor = -3 - 1 = -4$). In other words, the floor function just rounds down to the nearest integer. Thus by definition $\lfloor r \rfloor$ is an integer and $\lfloor r \rfloor \leq r$. Notice that if $\lfloor r \rfloor + 1 \leq r$, then $\lfloor r \rfloor$ is not the largest integer less than or equal to r (contradiction). Therefore, $r < \lfloor r \rfloor + 1$ and so $n = \lfloor r \rfloor$ is an integer such that $n \leq r < n+1$. ■