Homework #7:
Homework Dressed Up as a Paper

Billium Cook

October 7, 2020

Abstract

In this homework set, we look at some problems from Chapter 4. We also practice formatting something like a research paper, include a picture, and cite a source. This homework set is **DUE: Wednesday, October 14th**.

1 Our First Problem

I want this homework typed up (obviously) using LaTeX. You should (eventually) delete most of the garbage I have provided in this template. Please make sure you...

- Give your "paper" a better title.[1]
- Change the author name.
- Write an *brief* abstract summing up what is done in this homework set.
- Delete any template stuff that isn't relevant. For example, you shouldn't include my picture of "Obviously Isaac Newton".
- You should have **two** main sections. Section one (you come up with a better title) should address our first problem. Section two should have a bunch of subsections, each one addressing the next homework problem.

Problem #1: Look up a biography for someone famous from analysis. For example, Karl Weierstrass, Augustin-Louis Cauchy, and Richard Dedekind are all good examples of nineteenth century analysts. You can pick on of these guys or someone else if you wish.

I would like you to write a *very* brief bio-sketch (just a paragraph or two will do) and include a picture. Please *CITE YOUR SOURCE(S)!* Also, please include a relevant picture (either of that mathematician or some thing or place related to that person). I have included my picture of Sir Isaac Newton (well not quite) below.

[1] LaTeX note: double quotes always give right quotes " to get left quotes use ` – the key next to 1 on most keyboards.

1.1 A Subsection of Template Code

Citations in LaTeX are pretty easy as well. First, add a command to the bibliography section at the end of the tex file and then cite it as follows: This is some random textbook that I'm citing for no good reasons [BCW1899]. If you look at the code, I used "`\cite{bcw}`". Down in the bibliography notice that I tagged that entry with "`{bcw}`".

Including pictures is easy. Just upload the jpeg, png, bmp, or some other format (many work) to your Overleaf project or put the picture in the same folder as your tex file if your using a local compiler. The command to include the picture is `\includegraphics` (see this tex source for details).

Figure 1. Obviously, this is Sir Isaac Newton.

Again, here I randomly refer to my bibliography entries. For example, Sir Isaac Newton had strong fast food preferences as put forth in [N2020]. You might be interesting in checking out another questionable resource like [BCW1899].

1.2 Other Things You Might Want

Here's a sample theorem:

Theorem 1.1. *My ground breaking result*

$$1 + 1 = 2 \tag{1.1}$$

Proof: This is left to the reader as an exercise. ♦

There are other related commands: thm = Theorem, prop = Proposition, coro = Corollary, lem = Lemma, exa = Example, rem = Remark, de = Definition, hy = Hypothesis, or you can define your own.

If you look at the code, I have periodically used a `\label` command. These go with `\ref` commands to refer to those labels. For example: see Equation 1.1 or some Theorem 1.1 or Section 1.2.

One can create some rather amazing diagrams and graphs in LaTeX itself. Here's an example of such a diagram written using the "Tikz" package.

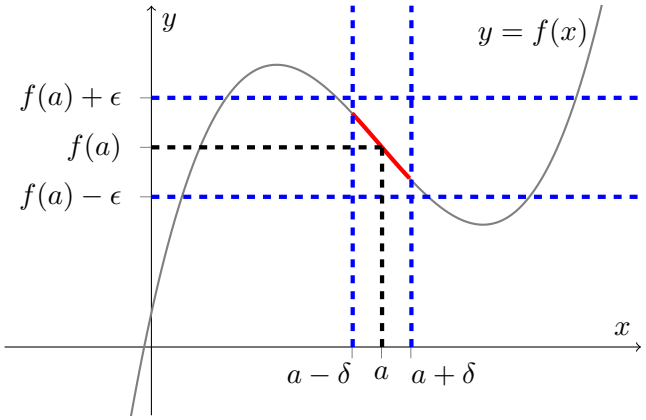

Figure 2. Epsilon-Delta Continuity at $x = a$.

2 Problems #2–5

The second section of your homework should include carefully typed up proofs of the following problems. You should change what is written **here**. ☺

2.1 Problem #2

In this problem we blah blah blah.

Proposition 2.1. *The sequence* $\left\langle n^3 - 10 \right\rangle_{n=0}^{\infty}$ *is bounded below but not above.*

Proof: This is left to the student as an exercise. ♦

Corollary 2.2. *The sequence* $\left\langle n^3 - 10 \right\rangle_{n=0}^{\infty}$ *diverges.*

Proof: This is left to the student as an exercise. ♦

2.2 Problem #3

In this problem we blah blah blah.

Proposition 2.3. *The sequence* $\left\langle \dfrac{6n^3 - 2n + 3}{2n^3 + n^2 - 1} \right\rangle_{n=1}^{\infty}$ *converges to* 3.

Proof: This is left to the student as an exercise. ♦

2.3 Problem #4

In this problem we blah blah blah.

Proposition 2.4. *The sequence* $\left\langle \dfrac{7n^2 + (-1)^n n}{2n^3 + 5n - \sin(n)} \right\rangle_{n=1}^{\infty}$ *converges to* 0.

Proof: This is left to the student as an exercise. ♦

2.4 Problem #5

In this problem we blah blah blah.

Proposition 2.5. *If* $\langle a_n \rangle \to 0$ *and* $\langle b_n \rangle \to 0$*, then* $\langle a_n b_n \rangle \to 0$.

Proof: This is left to the student as an exercise. ♦

2.5 Template Code and Example Problem to Be Deleted

In this subsection – which you should delete – we present an example convergence proof.

Proposition 2.6. *The sequence* $\left\langle \dfrac{10n^3 + 16n^2 + (-1)^n n + 3\cos(n)}{5n^3 - n + 2} \right\rangle_{n=1}^{\infty}$ *converges to* 2.

Proof: Suppose $\epsilon > 0$. Let $N = \left\lceil \frac{11}{2\epsilon} \right\rceil$. Notice that since $\epsilon > 0$, $11/(2\epsilon) > 0$ so $N \geq 1$ (N is a valid positive integer). Next, suppose that $n \geq N$. Thus $n \geq \left\lceil \frac{11}{2\epsilon} \right\rceil \geq \frac{11}{2\epsilon}$. Thus $\frac{11}{2n} \leq \epsilon$.

$$\left| \frac{10n^3 + 16n^2 + (-1)^n n + 3\cos(n)}{5n^3 - n + 2} - 2 \right| = \left| \frac{10n^3 + 16n^2 + (-1)^n n + 3\cos(n)}{5n^3 - n + 2} - 2 \cdot \frac{5n^3 - n + 2}{5n^3 - n + 2} \right|$$

$$= \left| \frac{10n^3 + 16n^2 + (-1)^n n + 3\cos(n) - 10n^3 + 2n - 4}{5n^3 - n + 2} \right| = \frac{\left|16n^2 + (-1)^n n + 3\cos(n) + 2n - 4\right|}{|5n^3 - n + 2|}$$

$$= \frac{16n^2 + (-1)^n n + 3\cos(n) + 2n - 4}{5n^3 - n + 2}$$

We pause to note that we can drop the absolute value bars because for $n \geq 1$ we have $5n^3 - n + 2 > 0$ and $16n^2 + (-1)^n n + 3\cos(n) + 2n - 4 \geq 16n^2 - n - 3 + 2n - 4 = 16n^2 + n - 7 > 0$. Now we our calculation (we grow the numerator and shrink the denominator to grow the fraction). Specifically we can drop negative terms from the top and positive terms from the bottom. In addition, we can replace both $(-1)^n$ and $\cos(n)$ with 1 as an upper estimate.

$$\frac{16n^2 + (-1)^n n + 3\cos(n) + 2n - 4}{5n^3 - n + 2} < \frac{16n^2 + n + 3 + 2n}{5n^3 - n} = \frac{16n^2 + 3n + 3}{5n^3 - n}$$

Now notice that $3 \leq 3n \leq 3n^2$ and $-n^3 \leq -n$. Thus we have:

$$\frac{16n^2 + 3n + 3}{5n^3 - n} \leq \frac{16n^2 + 3n^2 + 3n^2}{5n^3 - n^3} = \frac{22n^2}{4n^3} = \frac{22}{4n} = \frac{11}{2n} \leq \epsilon$$

We have shown that for every $\epsilon > 0$ there is an $N > 0$ such that for all $n \geq N$, we have $\left| \frac{10n^3 + 16n^2 + (-1)^n n + 3\cos(n)}{5n^3 - n + 2} - 2 \right| < \epsilon$. This establishes our convergence result. ♦

References

[N2020] Newton, Isaac, *The Burger King is a Jerk.* Her Majesty's Royal Publishing Company. June 2020.

[BCW1899] Bolzano, B., A.L. Cauchy, K. Weierstrass. We are going to party like it is eighteen hundred and ninety-nine. Princeton Communications, 5: 1982–1999.