

1. Consider the theorem " $A, \neg B \vdash_L \neg(A \rightarrow B)$ ".
  - (a) Write down the truth table for " $(A \wedge \neg B) \rightarrow \neg(A \rightarrow B)$ " to show it is a tautology.
  - (b) Suppose that in the middle of my "proof" of this proposition, I have a line which says " $\neg B \rightarrow (A \rightarrow B)$ ". How do I know there is a mistake in my proof?
2. My brother claims to have shown " $A \vdash_L A \rightarrow B$ " in proof system  $L$ . Could he possibly have found a proof? Explain why or why not.
3. Consider Theorem L14 (on page 20) which says " $\vdash_L A \rightarrow ((A \rightarrow B) \rightarrow B)$ ".
  - (a) Show this proposition is a tautology by writing down its truth table (an abbreviated truth table is ok).
  - (b) Prove this proposition in  $L$  using the deduction theorem and modus ponens (no lemmas allowed).
  - (c) Prove this proposition in  $L$  without the deduction theorem (you may use any lower numbered theorems).  
*Annoying hint:* Using previous lemmas, there is a 2 line proof.  
OR *Bullet proof/brute force method:* Use our translation procedures to turn (b)'s answer into (c)'s answer.
4. Prove at least 3 of the 7 theorems L19 through L25 (pages 21 and 22) – skip any one I do in class. You may use the deduction theorem and theorems with lower numbers (including ones you've skipped) in your proofs.