**Math 2110-410** **Homework #3** **Due: Wed., Feb. $12^{\text{th}}$, 2020**

1. Use induction to show that $1 + 3 + \cdots + (2n - 1) = n^2$ for all integers $n \geq 1$.
   *Note:* Use LaTeX to type up problem 1.
2. Prove that 8 is a factor of $9^n - 1$ for all natural numbers $n$.
3. Prove that $n^2 + n$ is always even [when $n$ is an integer].
   *Note:* Use LaTeX to type up problem 3.
4. Prove Theorem 20 (page 80): If no natural number $m$ such that $1 < m \leq \sqrt{p}$ divides $p$, then $p$ is prime.
5. Prove Theorem 34 (page 82): The sum of any three consecutive integers is divisible by 3.

Of course, you could type up all of your solutions, but I'm only requiring that you do this for problems 1. & 3.