Math 2110-410 # Homework #5 **Due: Fri., Mar. 20^{th}, 2020**

1. Let $A = \{15x + 6y \mid x, y \in \mathbb{Z}\}$ and $B = \{n \mid n \text{ is divisible by } 3\}$. Show $A = B$.

2. Determine what the following union and intersection are. Your answer should be of form: (a, b), $[a, b)$, $(a, b]$, or $[a, b]$ — that is an open, half-open, or closed interval.

   (a) $\displaystyle\bigcup_{n=1}^{\infty} \left(-1 + \frac{1}{n}, 1 + \frac{1}{n}\right)$

   (b) $\displaystyle\bigcap_{n=1}^{\infty} \left(-1 + \frac{1}{n}, 1 + \frac{1}{n}\right)$

3. For each of the following functions, determine if the function is one-to-one, onto, both or neither. Prove your answers.

   (a) $g_1 : \mathbb{R} \to \mathbb{R}_{\geq 0}$ defined by $g_1(x) = x^2 + 1$.

   (b) $g_2 : \mathbb{Z} \to \mathbb{Z}$ defined by $g_2(n) = \begin{cases} \dfrac{n}{2} & n \text{ is even} \\ 2n & n \text{ is odd} \end{cases}$.

   (c) $g_3 : \mathbb{Z} \to \mathbb{Z}$ defined by $g_3(n) = \begin{cases} n+1 & n \text{ is even} \\ 2n & n \text{ is odd} \end{cases}$.

4. Let $f : X \to Y$ and $g : Y \to Z$ be surjective functions. Prove that $g \circ f$ is surjective.

5. Let $f : X \to Y$ be a function, let $T_1, T_2 \subseteq Y$, and let $S_1, S_2 \subseteq X$. Prove the following:

   (a) $f(S_1) - f(S_2) \subseteq f(S_1 - S_2)$

   (b) $f^{-1}(T_1 \cup T_2) = f^{-1}(T_1) \cup f^{-1}(T_2)$

   (c) If $T_1 \subseteq T_2$, then $f^{-1}(T_1) \subseteq f^{-1}(T_2)$

Use $\LaTeX$ to type up problems 4 and 5.