Supplemental Problems: Jacobians & Change of Variables

1. Let R be the region bounded by $x-2y=1$, $x-2y=4$, $2x+y=1$, and $2x+y=3$. Compute $\iint_R \dfrac{x-2y}{2x+y}\,dA$ using the change of variables: $u=x-2y$ and $v=2x+y$.
2. Let R be the region bounded by $x+y=0$, $x+y=1$, $x-y=1$, and $x-y=4$. Compute $\iint_R (x-y)e^{x^2-y^2}\,dA$ using the change of variables: $u=x+y$ and $v=x-y$.
3. Let R be the region bounded by $x^2/16+y^2/4=1$. Compute $\iint_R e^{-x^2-4y^2}\,dA$. Hint: Use some sort of "elliptic" coordinates.
4. Let R be the region bounded by $x^2-y^2=1$, $x^2-y^2=4$, $x^2+y^2=9$, and $x^2+y^2=16$. Compute $\iint_R xy\,dA$ using the change of variables: $u=x^2-y^2$ and $v=x^2+y^2$.
5. Let R be the region bounded by $y=0$, $y=x$, and $x+y=\pi/4$ (a triangular region). Compute $\iint_R \dfrac{\sin(x-y)}{\cos(x+y)}\,dA$.
6. Let R be the region bounded by $y=x/2$, $y=0$, and $x+y=1$. Compute $\iint_R \sqrt{\dfrac{x+y}{x-2y}}\,dA$.
7. Let E be the region bounded by the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$. Compute $\iiint_E z^2\,dV$. Hint: Use some sort of "elliptic" type spherical coordinates.