Supplemental Problems: Lagrange Multipliers

For problems 1 – 10 find the minimum and maximum value of f (and where these values occur) subject to the given constraint. If a min or max does not exist, explain why not.

1. $f(x, y) = x^2 - y$ subject to $x^2 + y^2 = 25$
2. $f(x, y) = x - 3y - 1$ subject to $x^2 + 3y^2 = 16$
3. $f(x, y) = x^2 y$ subject to $x^2 + 2y^2 = 6$
4. $f(x, y) = \frac{1}{x} + \frac{1}{y}$ subject to $\frac{1}{x^2} + \frac{1}{y^2} = 1$
5. $f(x, y, z) = xyz$ subject to $x^2 + y^2 + z^2 = 1$
6. $f(x, y, z) = x^2 y^2 z^2$ subject to $x^2 + y^2 + z^2 = 1$
7. $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x^4 + y^4 + z^4 = 1$
8. $f(x, y) = x^2 + xy + y^2$ subject to $x^2 + y^2 \leq 4$
9. $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ subject to $x^2 + y^2 \leq 16$
10. $f(x, y) = e^{-xy}$ subject to $x^2 + 4y^2 \leq 1$
11. Find 3 numbers whose sum is 15 and product is a large as possible.
12. The temperature at coordinates (x, y) in Fake Desert, Utah is measured by the function $T(x, y) = 4x^2 - 4xy + y^2$. Frank is lost and traveling in a circle of with radius 5 centered at coordinates $(0, 0)$. What are the highest and lowest temperatures Frank experiences?
13. Find the point on the line $y = 2x + 3$ that is closest to $(4, 2)$.
14. Find the point on the plane $x + 2y + z = 1$ that is closest to the origin.
15. Find the point(s) on the surface $xy - z^2 = 1$ that are closest to the origin.
16. A rectangular package shipped U.S. Parcel Post cannot have a combined length and girth of more than 108 inches (girth is the perimeter of a cross-section perpendicular to the length). What dimensions maximize volume while satisfying this requirement?
17. Use Lagrange multipliers to show that of all rectangles with perimeter p, the square (with sides of length $p/4$) has the largest area.
18. Find the maximum value of $f(x_1, x_2, \ldots, x_n) = \sqrt[n]{x_1 x_2 \cdots x_n}$ subject to the constraints $x_1, \ldots, x_n \geq 0$ and $x_1 + x_2 + \cdots + x_n = c$ (where c is some fixed positive real number). Use your max value to conclude that the geometric mean is never larger than the arithmetic mean that is...

$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}$$