

Name: \_\_\_\_\_

Be sure to show your work!

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v} \quad \mathbf{r}''(t) = \left( \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|} \right) \mathbf{T}(t) + \left( \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|} \right) \mathbf{N}(t)$$

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho ds \quad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m} \left( \int_C x \rho ds, \int_C y \rho ds, \int_C z \rho ds \right)$$

$$\kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (14 points) Vector Basics: Let  $\mathbf{v} = \langle -1, -3, 1 \rangle$  and  $\mathbf{w} = \langle 1, 1, 2 \rangle$ .

(a) Find the vector of *length* 10 which points in the direction *opposite* that of  $\mathbf{v}$ .

(b) Find the angle between  $\mathbf{v}$  and  $\mathbf{w}$  (don't worry about evaluating inverse trig. functions).

Is this angle... **right**, **acute**, or **obtuse** ? (Circle your answer.)

(c) Match each expression with a corresponding statement describing what is being computed...

|                          |  |  |
|--------------------------|--|--|
| <input type="checkbox"/> | $\mathbf{a} \cdot \mathbf{b} = 0$                                    | <b>A)</b> the area of a parallelogram          |
| <input type="checkbox"/> | $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$                    | <b>B)</b> the vectors are orthogonal           |
| <input type="checkbox"/> | $ \mathbf{a} \times \mathbf{b} $                                     | <b>C)</b> nonsense                             |
| <input type="checkbox"/> | $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{b} \cdot \mathbf{c})$ | <b>D)</b> $\pm$ the volume of a parallelepiped |

2. (12 points) Let  $\ell_1$  be parametrized by  $\mathbf{r}_1(t) = \langle 1, 2, 0 \rangle + \langle -1, 3, 1 \rangle t$  and let  $\ell_2$  be the line which passes through the points  $P = (-2, 1, -3)$  and  $Q = (-1, 3, -1)$ . Determine if  $\ell_1$  and  $\ell_2$  are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

**3. (14 points)** A few points...

(a) Find the (scalar) equation of the plane through the points  $A = (1, 0, -1)$ ,  $B = (3, 2, 0)$ , and  $C = (2, 3, 1)$ .

(b) Parameterize the line which is perpendicular to the plane  $-5x + y + 2z = 13$  and which passes through the point  $P = (0, 2, -1)$ .

(c) Is the line found in part (b) orthogonal, parallel, both orthogonal and parallel, or neither orthogonal nor parallel to the plane found in part (a)? Explain.

**4. (10 points)** Let  $y = x^3 + 1$  where  $-1 \leq x \leq 2$ . Parameterize the graph of this function. Also, find its curvature.

**5. (16 points)** Let  $C$  be the helix parameterized by  $\mathbf{r}(t) = \langle 3 \cos(t), 5 \sin(t), 4 \cos(t) \rangle$ ,  $-\pi \leq t \leq \pi$ .

(a) Compute the **TNB**-frame for  $C$ .

(b) Find the curvature of  $C$ .

(c) Set up (but do **not** evaluate) the line integral  $\int_C (x^2 + z^2) e^y ds$  [Please simplify your answer.]

(d) Circle the correct answer:  $C$     IS    /    IS NOT    a planar curve.

**6. (10 points)** Suppose that a particle has a constant acceleration vector  $\mathbf{a}(t) = -4\mathbf{j}$ . Its initial velocity vector was  $\mathbf{v}_0 = 2\mathbf{i} - \mathbf{j}$  and its initial position was  $\mathbf{r}_0 = 100\mathbf{j}$ . Find a formula for the position of this particle,  $\mathbf{r}(t)$ , at time  $t$  (assume  $\mathbf{r}(t)$  is measured in meters and  $t$  in seconds).

What was the particle's initial **speed**? \_\_\_\_\_

**7. (14 points)** Let  $C$  be parameterized by  $\mathbf{r}(t) = \langle t, t^3, e^t \rangle$  where  $0 \leq t \leq 6\pi$ .

(a) Compute the curvature of  $C$ .

(b) Find the tangential and normal components of acceleration.

$$a_T = \underline{\hspace{4cm}}$$

$$a_N = \underline{\hspace{4cm}}$$

(c) Set up (but do **not** evaluate) the line integral  $\int_C (x^2y + z) \, ds$

**8. (10 points)** No numbers here. Choose **ONE** of the following:

I. Compute and simplify  $\frac{d}{dt} [\mathbf{r} \bullet (\mathbf{r}' \times \mathbf{r}'')]$  where  $\mathbf{r}(t)$  is a smooth vector valued function.

II. It is true that  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}|$ ? Either explain why this always holds or explain what would make it hold or fail.