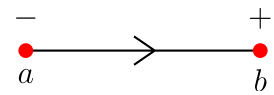
A Summary of our Big Theorems

Note: We make a blanket assumption that regions have piecewise smooth boundaries and all functions and/or components of vectors fields are sufficiently continuously differentiable on appropriate open sets. In other words, all of the objects involved are reasonably *nice*.

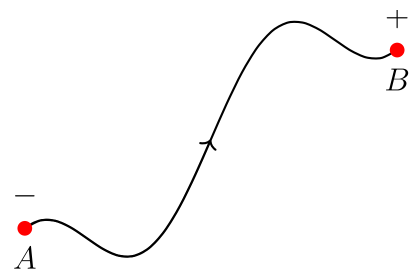
Fundamental Theorem of Calculus Let $I = [a,b]$ then the boundary of this interval is $\partial I = \{a, b\}$ where we are leaving a so it is negatively oriented and we are entering b so it is positively oriented. Thus...

$$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$$

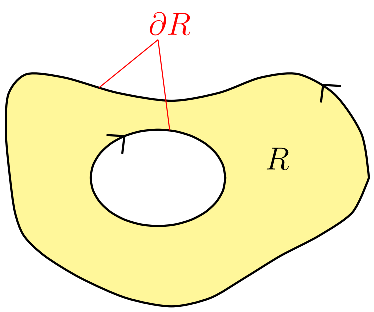


Fundamental Theorem of Line Integrals Let C be a curve whose endpoints are $\partial C = \{A, B\}$ where we are leaving A so it is negatively oriented and we are entering B so it is positively oriented. Thus...

$$\boxed{\int_C \nabla f \bullet d\mathbf{r} = f(B) - f(A)}$$



Green's Theorem(s)



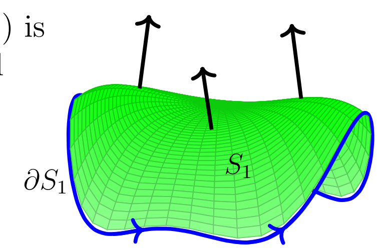
Recall that the boundary of R (i.e., ∂R) is oriented so that someone walking on top of R along the edge always has R to their left.

Work Version: $$\boxed{\iint\limits_R (N_x - M_y)\,dA = \int_{\partial R} M\,dx + N\,dy}$$

Flux Version: $$\boxed{\iint\limits_R (M_x + N_y)\,dA = \int_{\partial R} M\,dy - N\,dx}$$

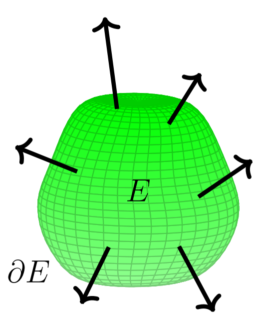
Stokes' Theorem Again, recall that the boundary of S_1 (i.e., ∂S_1) is oriented so that someone walking along edge on top of S_1 will always have S_1 to their left.

$$\boxed{\iint\limits_{S_1} (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,d\sigma = \int_{\partial S_1} \mathbf{F} \bullet d\mathbf{r}}$$



Divergence Theorem Let E be a solid region whose boundary (surface) ∂E is oriented outward (i.e., away from E).

$$\boxed{\iiint\limits_E \nabla \bullet \mathbf{F}\,dV = \iint_{\partial E} \mathbf{F} \bullet \mathbf{n}\,d\sigma}$$



Generalized Stokes' Theorem When M is an n-dimensional oriented manifold with boundary ∂M (which is an $(n-1)$-dimensional manifold with orientation induced from M) and ω is a differential $(n-1)$-form, then...

$$\boxed{\int_M d\omega = \int_{\partial M} \omega}$$