Math 2130

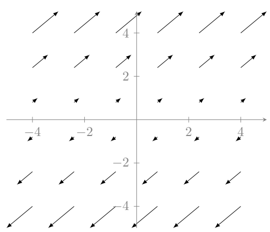
Vector Fields

Supplemental Homework

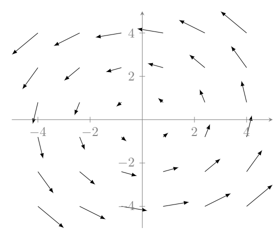
I. Sketch a plot of the following vector fields.

#1. $\mathbf{F}(x,y) = y^2\mathbf{i} + y\mathbf{j}$	#2. $\mathbf{F}(x,y) = x\mathbf{i} + \mathbf{j}$	#3. $\mathbf{F}(x,y) = \mathbf{i} + y\mathbf{j}$
#4. $\mathbf{F}(x,y) = y\mathbf{i} + y\mathbf{j}$	#5. $\mathbf{F}(x,y) = x\mathbf{i} - x\mathbf{j}$	#6. $\mathbf{F}(x,y) = y\mathbf{i} + x\mathbf{j}$
#7. $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$	#8. $\mathbf{F}(x,y) = x\mathbf{i} + y\mathbf{j}$	#9. $\mathbf{F}(x,y) = -x\mathbf{i} - y\mathbf{j}$

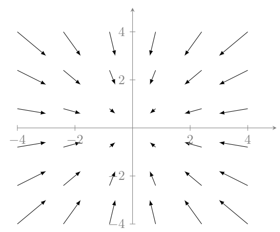
II. The following are plots of vector fields #1–#9 (listed above). Please note that all of the vectors have been scaled down so they do not overlap each other (i.e., relative but not absolute lengths of vectors are correct). Match the plot with its formula.



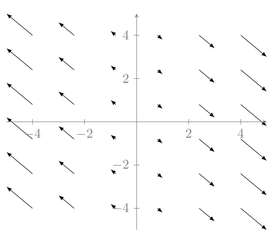
A



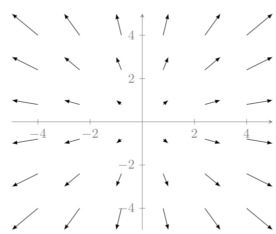
B



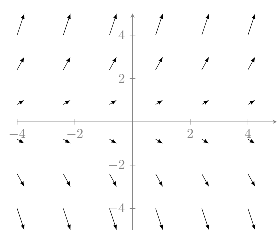
C



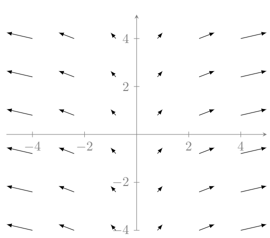
D



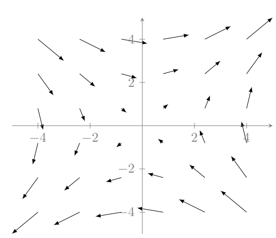
E



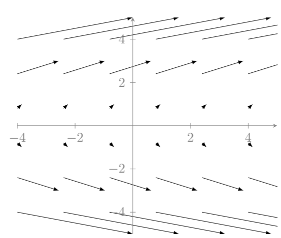
F



G



H



I

III. Of the vector fields above, which ones are conservative? If conservative, find all possible potential functions.

Answer Key: [Yes or No if conservative or not – here we check if $M_y = N_x$ when $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$. If Yes, the potential, $f(x,y)$, is given with arbitrary constant C.]

#1 = I. $\mathbf{F} = y^2\mathbf{i} + y\mathbf{j}$, No: $0 \neq 2y$

#2 = G. $\mathbf{F} = x\mathbf{i} + \mathbf{j}$, Yes: $0 = 0$, $f(x,y) = \dfrac{x^2}{2} + y + C$

#3 = F. $\mathbf{F} = \mathbf{i} + y\mathbf{j}$, Yes: $0 = 0$, $f(x,y) = x + \dfrac{y^2}{2} + C$

#4 = A. $\mathbf{F} = y\mathbf{i} + y\mathbf{j}$, No: $1 \neq 0$

#5 = D. $\mathbf{F} = x\mathbf{i} - x\mathbf{j}$, No: $0 \neq -1$

#6 = H. $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$, Yes: $1 = 1$, $f(x,y) = xy + C$

#7 = B. $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$, No: $-1 \neq 1$

#8 = E. $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$, Yes: $0 = 0$, $f(x,y) = \dfrac{x^2}{2} + \dfrac{y^2}{2} + C$

#9 = C. $\mathbf{F} = -x\mathbf{i} - y\mathbf{j}$, Yes: $0 = 0$, $f(x,y) = -\dfrac{x^2}{2} - \dfrac{y^2}{2} + C$