Math 2130 # Quiz #6 – Take Home **Due: Mon., Nov. 14^{th}, 2022**

Name: ______________________________

#1 Find the volume inside the ellipsoid: $\dfrac{(x-x_0)^2}{a^2}+\dfrac{(y-y_0)^2}{b^2}+\dfrac{(z-z_0)^2}{c^2}=1$ where $a,b,c>0$ and (x_0,y_0,z_0) is the center of the ellipsoid. Use the change of coordinates:

$$x=a\rho\cos(\theta)\sin(\phi)+x_0, \quad y=b\rho\sin(\theta)\sin(\phi)+y_0, \quad z=c\rho\cos(\phi)+z_0.$$

[This is a slightly modified version of spherical coordinates.]

You'll need to I) Compute the Jacobian $J=\dfrac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}$, II) Figure out what the ellipsoid's equation becomes in these new coordinates (the answer is very very simple), and III) Compute $\iiint_E 1\,dV$ using I and II.

#2 Find the centroid of the quarter annulus R: $1\le x^2+y^2\le 4$ in the first quadrant (i.e., $x,y\ge 0$).

#3 Consider the region E bounded by $z=1$ and $z=10-x^2-y^2$. Find the centroid.