DUE: Monday, October 6^{th} at the **beginning** of class.

I. From section 3.5, do problems 11, 37, and 38 – sketch a nice picture to help explain the system of equations in #38.

II. An $n \times n$ matrix N is said to be *nilpotent* if $N^k = 0_{n\times n}$ for some positive integer k.

(a) Verify that $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is nilpotent.

(b) Use the determinant to prove that nilpotent matrices are singular.

(c) Let $\mathbf{v}$ be an eigenvector with eigenvalue λ for a matrix A. By definition, $A\mathbf{v} = \lambda\mathbf{v}$. What is $A^2\mathbf{v} =$? or in general, for $\ell \geq 0$, what is $A^\ell\mathbf{v} =$?

(d) Find the eigenvalues and eigenvectors of the matrix B (from part (a)).

(e) Prove that the only eigenvalue of a nilpotent matrix is zero (*Hint:* use part (c) and $N^k = 0$).