MATH 2240	VECTOR SPACES	PROJECT #4

DUE: Monday, November 3^{rd} at the **beginning** of class.

I. From section 4.8, do problems 38 and 40 – Use Maple to plot these equations in both the original and rotated coordinates.
Note: Use `implicitplot` to plot these conic sections. For example:

$$\left[\right\rangle \quad \text{with(plots)} :$$

$$\left[\right\rangle \quad \text{implicitplot}\left(x^2 + \frac{y^2}{4} = 1, x = -2..2, y = -2..2, \text{scaling} = \text{constrained}\right);$$

II. Let U and W be subspaces of a vector space V.

(a) Show that $U \cap W = \{v \in V \mid v \in U \text{ and } v \in W\}$ is a subspace of V.

(b) Show that $U + W = \{u + w \mid u \in U \text{ and } w \in W\}$ is a subspace of V.

BONUS: Prove that $\dim(U + W) = \dim(U) + \dim(W) - \dim(U \cap W)$

III. Consider $V = P_3$ (polynomials of degree 3 and less). Let $U = \{f(x) \in P_3 \mid f(0) = 0\}$ and let $W = \{f(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \mid a_3 + a_2 + a_1 + a_0 = 0\}$.

(a) Show that U and W are subspaces.

(b) Show that $\beta = \{t, t^2, t^3\}$ is a basis for U.

(c) Find a basis W (remember to show that your basis *is* a basis).

(d) Find a basis for $U \cap W$.

(e) Show that $U + W = P_3$.