MATH 2240	EIGEN-PROJECT	PROJECT #5

DUE: Tuesday, November 25^{th} by noon.

I. Let $T : P_2 \to P_2$ be defined by $T(f(t)) = f''(t) + 2f'(t) + f(t)$. Also, let $\alpha = \{1, t, t^2\}$ (the standard basis). Note: T is a linear transformation.

(a) Determine $[T]_\alpha$.

(b) Find bases for both the Kernel and Range of T. In addition, find the nullity and rank of T. Is T invertible?

(c) Find the eigenvalues of T. Find the cooresponding eigenvectors.

(d) Determine all geometric and algebraic multiplicities. Is T diagonalizable?

II. Let $A = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

(a) Find eigenvectors and eigenvalues for A.

(b) Find a matrix, P, which diagonalizes A.

(c) Use part (b) to find $B = \sqrt{A}$ (that is: $B^2 = A$).

III. Let A be an $n \times n$ matrix.

(a) Show that A is non-singular (i.e. invertible) if and only if $\lambda = 0$ is not an eigenvalue of A.

(b) Show that A and A^{-1} have the same eigenvectors.

(c) How are the eigenvalues of A and A^{-1} related? Give an example. Then prove your conjectured relationship in general.