Math 2240 **Homework #2** **Due: Tues., July 21^{st}, 2020**

Be sure to show your work!

Notes:

- This first homework was shorter. These later homeworks are longer. I don't plan on weighting each homework equally.
- You can copy/paste my sample code into Maple. However, if you would prefer to do computations using different software, that is ok. Make sure you turn in *all* of your work (including computer code).
- If you end up doing the entire assignment in Maple, feel free to turn in a Maple worksheet (.mw file) instead of a pdf.
- **While some calculations can be done by hand**, some are better done using technology. A calculator is a poor choice since I want *exact* answers not decimal approximations.

1. Consider $W = \text{span}\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{u}, \mathbf{v}, \mathbf{w}\}$ (a subspace of $\mathbb{R}^{6\times 1}$) where $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{u}, \mathbf{v}, \mathbf{w}$ are the vectors defined in Maple as follows:

```
restart;
with(LinearAlgebra):
a := <<1,2,1,3,1,2>>:
b := <<2,-1,-2,0,1,-1>>:
c := <<-2,-4,-2,-6,-2,-4>>:
u := <<-4,7,8,6,-1,7>>:
v := <<1,1,-1,3,2,1>>:
w := <<2,-3,0,-6,-3,-3>>:
"[ a | b | c | u | v | w ]" = <a|b|c|u|v|w>;
```

(a) Choose vectors from the list: $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{u}, \mathbf{v}, \mathbf{w}$ that give us a basis β for W.

(b) Take your basis β from part (a) and extend it to a basis for all of $\mathbb{R}^{6\times 1}$ (i.e., find a basis for $\mathbb{R}^{6\times 1}$ that includes the vectors in β).

(c) Given the following vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ (defined below), which vectors belong to W?

```
x := <<6,5,4,3,2,1>>:
y := <<1,-13,-11,-15,-2,-13>>:
z := <<13,-3,-3,-9,2,-11>>:
"[ x | y | z ]" = <x|y|z>;
```

2. Let $W = \text{Span}(S) \subseteq \mathbb{P}_3$ where $S = \{t^3 + 2t - 1, -2t^3 - 4t + 2, t^3 + 2t^2 + 3t + 1, t^3 - 4t^2 - 5\}$.

(a) Find a basis α for W.

(b) Decide if any of $f(t) = t^3 + t$, $g(t) = t^3 - 2t^2 + t - 3$, or $h(t) = 2t^2 + 3$ belong to W.
For those that belong to W, find their α-coordinate vectors.

(c) Extend α to a basis β for $\mathbb{P}_3$.

(d) Suppose that $[p(t)]_\beta = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. What is $p(t)$?

3. Let $\alpha = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ and $\beta = \left\{ \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$.

(a) Show that both α and β are bases for $\mathbb{R}^{2\times 2} = M_{2\times 2}$.

(b) Find the change of coordinate matrix $P = \mathcal{P}_{\beta\leftarrow\alpha} = [I]_\alpha^\beta$ (changing from α- to β-coordinates).

(c) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find $[A]_\alpha$. Then use part (b) to compute $[A]_\beta$.