# Propositional Logic: Proofs in L

## Axioms:

**Axiom 1:** $A \to (B \to A)$

**Axiom 2:** $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

**Axiom 3:** $(\neg B \to \neg A) \to ((\neg B \to A) \to B)$

## Rules of Inference:

**Modus Ponens:** If $A$ and $A \to B$ are lines in a proof, we can write $B$ as a (later) line. Here $A$ and $B$ can represent any formulas.

## Shortcuts:

**Lemmas:** You may write an instance of a previously proved (lowed numbered) theorem as a line, if the instances of the "givens" of that theorem appear as earlier lines.

**Deduction Theorem:** Instead of proving $G_1, \ldots, G_\ell \vdash A \to B$, prove $G_1, \ldots, G_\ell, A \vdash B$.

## Abbreviations:

**Conjunction:** $A \wedge B$ abbreviates $\neg(A \to \neg B)$

**Disjunction:** $A \vee B$ abbreviates $(\neg A) \to B$

**Double Implication:** $A \leftrightarrow B$ abbreviates $\neg((A \to B) \to \neg(B \to A))$
*Note:* When $A \leftrightarrow B$ and $(A \to B) \wedge (B \to A)$ are unabbreviated, they are identical.

## Theorems:

**L1:** $\vdash_L A \to A$

**L2:** $\vdash_L (\neg B \to B) \to B$

**L3:** $A \to (B \to C),\ A \to B \vdash_L A \to C$

**L4:** $A \to ((B \to A) \to C) \vdash_L A \to C$

**L5:** $B \vdash_L A \to B$

**L6:** $A \to (B \to C),\ B \vdash_L A \to C$

**L7:** $A \to (B \to C) \vdash_L B \to (A \to C)$

**L8:** $A \to B,\ B \to C \vdash_L A \to C$

**L9:** $P \to R \vdash_L P \to (Q \to R)$

**L10:** $\vdash_L (\neg B \to \neg A) \to (A \to B)$

**L11:** $\vdash_L \neg\neg B \to B$

**L12:** $\vdash_L B \to \neg\neg B$

**L13:** $\vdash_L (A \to (B \to C)) \to (B \to (A \to C))$

**L14:** $\vdash_L A \to ((A \to B) \to B)$

**L15:** $\vdash_L \neg A \to (A \to B)$

**L16:** $\vdash_L (A \to B) \to (\neg B \to \neg A)$

**L17:** $\vdash_L A \to (\neg B \to \neg(A \to B))$

**L18:** $A,\ B \vdash_L A \wedge B$

**L19:** $A \wedge B \vdash_L A$

**L20:** $A \wedge B \vdash_L B$

**L21:** $A \vdash_L A \vee B$

**L22:** $B \vdash_L A \vee B$

**L23:** $A \leftrightarrow B \vdash_L A \to B$

**L24:** $A \leftrightarrow B \vdash_L B \to A$

**L25:** $A \to B, B \to A \vdash_L A \leftrightarrow B$