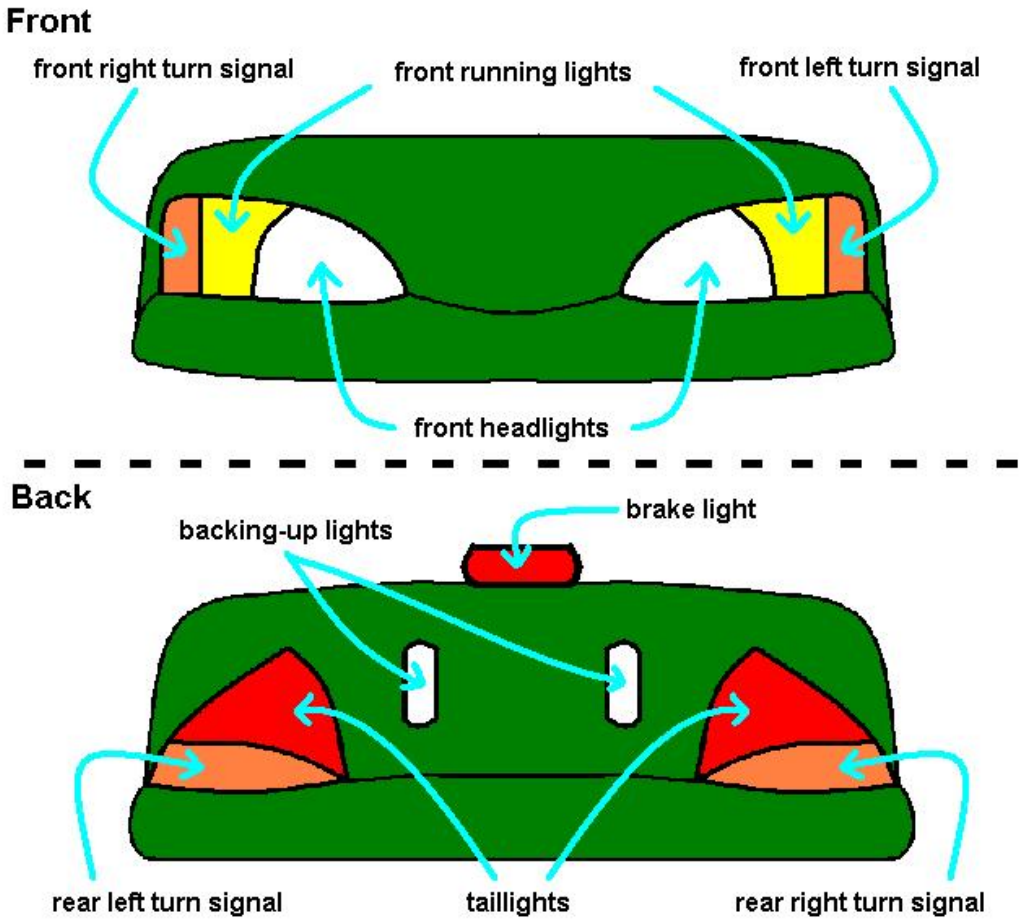
Automobile Light Propositions

This is an example of how one might go about characterizing the operation of exterior lights on an automobile in logic. In particular this is a logical model of the operation of the exterior lights of my own car – a 2002 Nissan Sentra. For simplicity, I have ignored the distiction between high and low beams (both the headlights and the taillights have three states: "off", "on", and "on bright").



My car's exterior lights.

1 Atomic Propositions

Let us begin by listing the atomic propositions along with their intended meanings. First the atomic propositions associated with the lights themselves.

- Headlights(on) is true if and only if the vehicle's headlights are on.
- Headlights(off) is true if and only if the vehicle's headlights are off.
- FrontRunningLights(on) is true if and only if the vehicle's front running lights are on.
- FrontRunningLights(off) is true if and only if the vehicle's front running lights are off.
- Flasher(front, left, on) is true if and only if the vehicle's front left flasher is on.

- Flasher(front, left, off) is true if and only if the vehicle's front left flasher is off.
- Flasher(front, left, flashing) is true if and only if the vehicle's front left flasher is blinking on and off.
- Flasher(front, right, on) is true if and only if the vehicle's front right flasher is on.
- Flasher(front, right, off) is true if and only if the vehicle's front right flasher is off.
- Flasher(front, right, flashing) is true if and only if the vehicle's front right flasher is blinking on and off.
- Flasher(rear, left, on) is true if and only if the vehicle's rear left flasher is on.
- Flasher(rear, left, off) is true if and only if the vehicle's rear left flasher is off.
- Flasher(rear, left, flashing) is true if and only if the vehicle's rear left flasher is blinking on and off.
- Flasher(rear, right, on) is true if and only if the vehicle's rear right flasher is on.
- Flasher(rear, right, off) is true if and only if the vehicle's rear right flasher is off.
- Flasher(rear, right, flashing) is true if and only if the vehicle's rear right flasher is blinking on and off.
- Taillights(on) is true if and only if the vehicle's taillights are on.
- Taillights(off) is true if and only if the vehicle's taillights are off.
- BrakeLight(on) is true if and only if the vehicle's brake light is on.
- BrakeLight(off) is true if and only if the vehicle's brake light is off.
- BackingUpLights(on) is true if and only if the vehicle's backing-up lights are on.
- BackingUpLights(off) is true if and only if the vehicle's backing-up lights are off.

Next let's list the atomic propositions associated with the various switches.

- Ignition(on) is true if and only if the car's ignition is turned on. I will assume that if the ignition is on then the car is *actually* running (unlike my last car).
- Ignition(off) is true if and only if the car's ignition is turned off.
- Shifter(reverse) is true if and only if the car is in reverse.
- BrakePedal(depressed) is true if and only if the brake pedal is depressed.
- BrakePedal(released) is true if and only if the brake pedal is not depressed.
- TurnSignal(left) is true if and only if the turn signal is signaling left.
- TurnSignal(right) is true if and only if the turn signal is signaling right.
- TurnSignal(off) is true if and only if the turn signal is not signaling.
- EmergencyLights(on) is true if and only if the hazard light switch is on.
- EmergencyLights(off) is true if and only if the hazard light switch is off.
- LightSwitch(headlights) is true if and only if the headlights are switched on.
- LightSwitch(running) is true if and only if the running lights are switched on.
- LightSwitch(off) is true if and only if the headlights and running lights are switched off.

2 Each object must be in some state.

First we must guarantee that each light, switch, signal, or pedal occupies some valid state.

- $(\text{Headlights(on)} \vee \text{Headlights(off)}) \wedge \neg(\text{Headlights(on)} \wedge \text{Headlights(off)})$
  This says that the headlights must be either on or off and they cannot be both on and off.
- $(\text{FrontRunningLights(on)} \vee \text{FrontRunningLights(off)}) \wedge$
  $\neg(\text{FrontRunningLights(on)} \wedge \text{FrontRunningLights(off)})$
  This says that the front running lights must be either on or off and they cannot be both on and off.
- $(\text{Flasher(front, left, on)} \vee \text{Flasher(front, left, off)} \vee \text{Flasher(front, left, flashing)}) \wedge$
  $\neg(\text{Flasher(front, left, on)} \wedge \text{Flasher(front, left, off)}) \wedge$
  $\neg(\text{Flasher(front, left, on)} \wedge \text{Flasher(front, left, flashing)}) \wedge$
  $\neg(\text{Flasher(front, left, flashing)} \wedge \text{Flasher(front, left, off)})$
  This says that the front left flasher must be either on, flashing, or off but not some combination of these states.
- $(\text{Flasher(front, right, on)} \vee \text{Flasher(front, right, off)} \vee \text{Flasher(front, right, flashing)}) \wedge$
  $\neg(\text{Flasher(front, right, on)} \wedge \text{Flasher(front, right, off)}) \wedge$
  $\neg(\text{Flasher(front, right, on)} \wedge \text{Flasher(front, right, flashing)}) \wedge$
  $\neg(\text{Flasher(front, right, flashing)} \wedge \text{Flasher(front, right, off)})$
  This says that the front right flasher must be either on, flashing, or off but not some combination of these states.
- $(\text{Flasher(rear, left, on)} \vee \text{Flasher(rear, left, off)} \vee \text{Flasher(rear, left, flashing)}) \wedge$
  $\neg(\text{Flasher(rear, left, on)} \wedge \text{Flasher(rear, left, off)}) \wedge$
  $\neg(\text{Flasher(rear, left, on)} \wedge \text{Flasher(rear, left, flashing)}) \wedge$
  $\neg(\text{Flasher(rear, left, flashing)} \wedge \text{Flasher(rear, left, off)})$
  This says that the rear left flasher must be either on, flashing, or off but not some combination of these states.
- $(\text{Flasher(rear, right, on)} \vee \text{Flasher(rear, right, off)} \vee \text{Flasher(rear, right, flashing)}) \wedge$
  $\neg(\text{Flasher(rear, right, on)} \wedge \text{Flasher(rear, right, off)}) \wedge$
  $\neg(\text{Flasher(rear, right, on)} \wedge \text{Flasher(rear, right, flashing)}) \wedge$
  $\neg(\text{Flasher(rear, right, flashing)} \wedge \text{Flasher(rear, right, off)})$
  This says that the rear right flasher must be either on, flashing, or off but not some combination of these states.
- $(\text{Taillights(on)} \vee \text{Taillights(off)}) \wedge \neg(\text{Taillights(on)} \wedge \text{Taillights(off)})$
  This says that the taillights must be either on or off and they cannot be both on and off.
- $(\text{BrakeLight(on)} \vee \text{BrakeLight(off)}) \wedge \neg(\text{BrakeLight(on)} \wedge \text{BrakeLight(off)})$
  This says that the brake light must be either on or off and it cannot be both on and off.
- $(\text{BackingUpLights(on)} \vee \text{BackingUpLights(off)}) \wedge$
  $\neg(\text{BackingUpLights(on)} \wedge \text{BackingUpLights(off)})$
  This says that the backing-up lights must be either on or off and they cannot be both on and off.
- $(\text{Ignition(on)} \vee \text{Ignition(off)}) \wedge \neg(\text{Ignition(on)} \wedge \text{Ignition(off)})$
  This says that the car is either turned on or off, and it cannot be both on and off.

- (BrakePedal(depressed) ∨ BrakePedal(released))∧
  ¬(BrakePedal(depressed) ∧ BrakePedal(released))
  This says that the brake pedal is either depressed or not depressed, and it cannot be both depressed and not depressed.
- (TurnSignal(left)∨TurnSignal(right)∨TurnSignal(off))∧¬(TurnSignal(left)∧TurnSignal(right))∧
  ¬(TurnSignal(left) ∧ TurnSignal(off)) ∧ ¬(TurnSignal(right) ∧ TurnSignal(off))
  This says that the turn signal must select either left, right, or off but not some combination of those states.
- (EmergencyLights(on) ∨ EmergencyLights(off))∧
  ¬(EmergencyLights(on) ∧ EmergencyLights(off))
  This says that the hazard lights be either on or off but they cannot be both on and off.
- (LightSwitch(headlights) ∨ LightSwitch(running) ∨ LightSwitch(off))∧
  ¬(LightSwitch(headlights) ∧ LightSwitch(running))∧
  ¬(LightSwitch(headlights) ∧ LightSwitch(off))∧
  ¬(LightSwitch(running) ∧ LightSwitch(off))
  This says that the lights must be switched to either headlights, running lights, or be switched off and they cannot be switched to some combination of those states.

Remark: You may notice that I have said nothing about Shifter(reverse). Since none of the other shifter states are present in this model (like for instance Shifter(park)), I don't need to specify that one of a list shifter states is chosen and no others.

3 Does the ignition matter?

Now we can finally get to describing how the car's exterior lights behave. It turns out that most lights operate the same whether the car's ignition is turned on or off. Among such lights are the brake light, emergency lights, running lights, and headlights. On the other hand, the turn signals and the backing-up lights do not work when the car is turned off.

- (BrakePedal(depressed) ↔ BrakeLight(on)) ∧ (BrakePedal(depressed) → Taillights(on))
  This says that the brake light comes on only when the brake pedal is depressed and that pressing the brake pedal will turn on the taillights.
  Note: I don't need to specify that when the brake pedal is released then the brake light goes off since if the brake light is on, then the brake pedal must be depressed and one of the previous propositions tells me that the brake pedal cannot be both depressed and released.
- EmergencyLights(on) → (Flasher(front, left, flashing) ∧ Flasher(front, right, flashing)∧
  Flasher(rear, left, flashing) ∧ Flasher(rear, right, flashing))
  The emergency light switch makes all four turn signals flash on and off.
- (LightSwitch(running)∨LightSwitch(headlights)) → (FrontRunningLights(on)∧Taillights(on))
  When either running lights or headlights are selected by the light switch, the front running lights and taillights come on.
- LightSwitch(headlights) ↔ Headlights(on)
  The headlights will only come on if the headlights are selected by the light switch.

- Taillights(on) → (BrakePedal(depressed)∨LightSwitch(running)∨LightSwitch(headlights))
  If the taillights are on, then either the brake pedal is depressed or the light switch has the running lights or headlights selected.
- FrontRunningLights(on) → (LightSwitch(running) ∨ LightSwitch(headlights))
  If the front running lights are on, then the light switch must be selecting running lights or headlights.

This takes care of the headlights, taillights, brake light, and running lights. Next we need to examine the turn signals.

4 Ignition on!

Let's now describe the behavior of the turn signals.

- ¬(Flasher(front, left, on) ∨ Flasher(front, right, on)∨
  Flasher(rear, left, on) ∨ Flasher(rear, right, on))
  My flashers are never continually on – they only flash.
- (Flasher(front, left, flashing) ↔ Flasher(rear, left, flashing))∧
  (Flasher(front, right, flashing) ↔ Flasher(rear, right, flashing))
  The front and rear flashers work together. The front left flasher will flash on and off if and only if the rear left flasher is flashing on and off. The same is true for the right flashers.
- (Ignition(on) ∧ TurnSignal(left)) → Flasher(front, left, flashing)
  When the car is on and we signal left, the front left flasher flashes on and off.
  Note: We don't need to specify that the rear left flasher flashes since one of the last rules says that front and rear flashers work together.
- (Ignition(on) ∧ TurnSignal(right)) → Flasher(front, right, flashing)
  When the car is on and we signal right, the front right flasher flashes on and off.
  Note: We don't need to specify that the rear right flasher flashes since one of the lasts rule says that front and rear flashers work together.
- Flasher(front, left, flashing) → (EmergencyLights(on) ∨ (Ignition(on) ∧ TurnSignal(left)))
  If the front left flasher is on, then either the hazard lights are activated or the car is on and signaling left.
- Flasher(front, right, flashing) → (EmergencyLights(on)∨(Ignition(on)∧TurnSignal(right)))
  If the front right flasher is on, then either the hazard lights are activated or the car is on and signaling right.

Note: My car ignores the turn signal if the emergency lights are already on.

Finally, I will describe the behavior of the backing-up lights.

- BackingUpLights(on) ↔ (Ignition(on) ∧ Shifter(reverse))
  The backing-up lights will come on if and only if the car is on and in reverse.