Math 2510: Introduction to Proof & Logic: Final Exam Projects

As we discussed in class, we will have final projects instead of a regular in class test. You will pick a topic related to, but yet, a little beyond what was covered in class. Then you will write up your findings and give a short presentation during the final exam period.

Here is what I expect from you (a what you will be graded on):

Project Proposal Pick out a topic and outline what you plan to study. Write up a brief description (not more than 1 page) of your project. This will be due in approximately 1 week. Everyone will meet with me to discuss their proposal and I will give you suggestions (possible examples, problems, theorems to consider).

The Paper Your paper will be due at the beginning of our final exam period (before you give your talk).

- The paper should be at least 5 pages long.
- Your classmates are your audience. Make sure your paper is understandable from their current level of mathematical education [build on what we covered in class].
- The first page should give some background on your topic. Include some history. If you were studying Hilbert spaces, you should indicate who first defined such a space and maybe give a brief biography of that mathematician. If your topic doesn't lend itself to such a discussion, pick a prominent mathematician associated with your topic and give a brief biography.
- Your paper should include 1 "big theorem" and its proof.
- You should give at least 3 "concrete" examples illustrating your topic.
- You should do at least 3 "homework problems" related to your topic. (These could be calculations or some easy-to-prove theorems – just think of something I might assign if we covered this in class.)
- Your paper should include a discussion of some application of your topic. This can be a scientific, "real-world", or mathematical application.
- You should do a inductive proof and proof by contradiction somewhere in the paper — if you can't come up with a reason to use these techniques, come talk to me and I'll help you make something up.

The Presentation Everyone will give a 10 minute presentation of what they learned during the final exam period. Sum up what you learned. Give some examples. Describe your big theorem and at least "sketch" its proof.

Point breakdown: Proposal 5%, Paper 70%, Presentation 25%.

Formatting:

Your paper **MUST** be typed. You may use whatever you wish to type it up — Word, Wordperfect, LaTeX, Maple Document, anything! Part of your grade will be based on how "nice" your paper looks – i.e. don't use Notepad. If you decide to use LaTeX (the best choice, of course), I will give you some extra points.

You are free to choose your mode of presentation. You may use slides or just write on the whiteboard. If you make a Powerpoint (or better yet LaTeX) slideshow, please let me know ahead of time so I can get a laptop and projector for our room. If you plan to use overhead slides, let me know so I can make sure we have an overhead projector during the exam period.

Random Topic Suggestions!!!

Here are some suggested topics. Your don't have to pick from this list, these are just random suggestions. By the way, if you pick a topic closely related to someone else's topic, please feel free to coordinate your papers and presentations — you still have to write up your own stuff, but can build off each other's background.

- Cauchy Sequences. A metric space is called "complete" if every Cauchy sequence converges. It can be proved that every metric space can be embedded in a complete metric space. The real numbers are complete in this sense.
- Uniform continuity — a powerful assumption.
- Normed spaces and inner-product spaces.
- Path connected. Every path connected space is connected. But the converse does not hold. Path connectedness is more commonly used than regular connectedness.
- Product and Box topologies. Given topologies defined on spaces X and Y one can define a topology on $X \times Y$. This can be done for infinite cartesian products as well. The product of compact spaces is still compact.
- Symmetric groups and group actions. One can easily classify the platonic solids. Each of these has a group of symmetries. The rotational symmetries of the icosahedron form the smallest non-abelian simple group.
- Quotients of groups, vector spaces, topological spaces. Gluing topological spaces together.
- De Rham cohomology and the generalized Stoke's theorem.
- Dual spaces and bounded linear operators. What does bounding an operator's norm have to do with continuity?
- Sizes of infinity and transfinite induction.
- Permutations and determinants
- Change of basis in linear algebra and Jordan form. Matrix exponential.
- Complex variables. Better than Taylor series it's....Laurent series! How the coefficient of x^{-1} computes integrals.